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ABSTRACT

This book is directed toward increasing students' ability to analyze problems and comprehend what they read and hear. It outlines and illustrates the methods that good problem solvers use in attacking complex ideas, and provides practice in applying these methods to a variety of questions involving comprehension and reasoning. Chapter I includes a test of analytical skills, and chapter II presents a posttest. Intervening chapters consider errors in reasoning, problem-solving methods, verbal reasoning problems, six myths about reading, analogies, writing relationship sentences, how to form analogies, analysis of trends and patterns, solving mathematical word problems, and a post test. Appendices present problems, solutions, and an answer key. (MNS)

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How to sharpen your thinking skills and increase your IQ

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THIRD EDITION

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Problem Solving and Comprehension

Third Edition

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PREFACE

Suppose you asked people the following questions: Would you like greater skill in solving math and logic problems? Would you like to sharpen your grasp of the ideas you read in scientific publications, medical reports, textbooks, and legal contracts?

Most people would answer "yes" to these questions. They'd be happy to gain some extra muscle in reasoning and comprehension—because in today's world it's almost impossible to avoid doing some problem solving and technical reading.

The business world and the classroom have always put a premium on mental skills. But today, even in the home, checkbooks and budgets need balancing, wits are challenged by directions for assembling toys and stereo equipment, income tax forms must be deciphered, and so on.

This book shows you how to increase your power to analyze problems and comprehend what you read and hear. First it outlines and illustrates the methods that good problem solvers use in attacking complex ideas. Then it gives you practice in applying these successful methods to a variety of questions in comprehension and reasoning. As you work through the book you will witness a steady improvement in your analytical thinking skills. You will develop confidence in your own ability to solve problems, and this increased confidence will give you a vigorous, positive attitude when attacking problems. If you're willing to work and practice, you'll be well rewarded.

For example, at some time you may have to take a test to enter college, medical school or law school—or to be hired for a particular job. Here are some of the tests commonly used for college and job selection:

- Scholastic Aptitude Test (SAT)
- Graduate Record Examination (GRE)
- Law School Admission Test (LSAT)
- Wonderlic Personnel Test
- United States Employment Service
  General Aptitude Test Battery
- Civil Service Examinations

Tests such as these are made up of problems; the better you are at problem solving the higher your scores will tend to be. If you put the techniques you learn in this book to work, especially in high school and college courses where you have a chance to use them vigorously, you can expect to see real gains in your scores on selection tests.

Your school grades can also be improved with the techniques you learn here because you will have a two-edged sword. First, you'll be better prepared to understand your textbooks and lectures so that your mastery of courses will be fuller and deeper. You'll be a better thinker and learner. In addition, when exam time rolls around, your sharpened reading and reasoning skills will give you a strong advantage in interpreting questions and answering them.

But the thinking skills you learn in this book go beyond tests and school learning. You'll find them useful in all occupations that involve reading technical materials or tackling difficult problems. With the growth of technology, such occupations represent an expanding portion of the job market. X-ray technicians, TV repairmen, registered nurses, computer programmers, and accountants are all called upon to comprehend and coordinate advanced areas of knowledge. Automobile repair has become a field of specialists. Modern carburetors are so intricate that a person can't understand and repair them without the ability to read complex descriptions and directions. And skilled operators are needed for the new automotive diagnostic equipment which is used to get a profile of engine performance and difficulties. Even farming today has become a detailed science in which soils are chemically analyzed and then treated with spectrums of additives to produce maximum yields of high-paying crops. To be successful a farmer has to have considerable knowledge about both chemical products and crop-market trends.

In short, the techniques you learn in this book can help you on tests, in your academic courses, and in any occupations which involve analyzing, untangling, or comprehending knotty ideas.

CONTENTS

I. Test Your Mind—See How It Works 1

II. Errors in Reasoning 11

III. Problem-Solving Methods 21

IV. Verbal Reasoning Problems 41

V. Six Myths About Reading 137

VI. Analogies 141

VII. Writing Relationship Sentences 155

VIII. How to Form Analogies 171

IX. Analysis of Trends and Patterns 193

X. Solving Mathematical World Problems 221

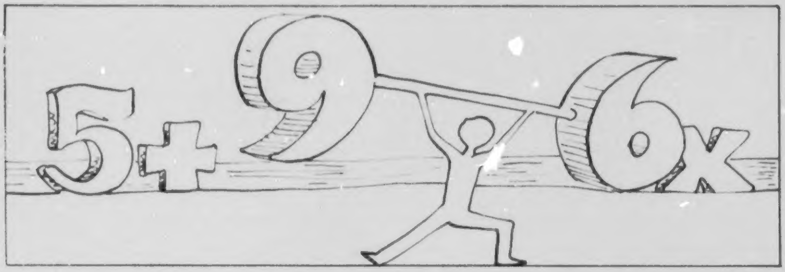
XI. The Post-Wasi Test 315

Appendix I 323

Appendix II 325

Appendix III 343

7



I. TEST YOUR MIND—SEE HOW IT WORKS

A good way to begin a thinking skills program is to take stock of your own thinking habits and compare them to those of other people. On the next page you'll find a test called the Whimbey Analytical Skills Inventory (WASI). The WASI is the type of test you might take in applying for a job or college program. If you are using this book in a class your instructor will ask you take the WASI and make an extra copy of your answers. Then he will collect one copy.

Here is how the WASI differs from other tests. Usually when you take such tests you don't get a chance to discuss your answers. Sometimes you don't even find out what your scores are. But with the WASI you will spend several days in class debriefing—going over the test item by item. For each question, your instructor will call on different students to explain how they handled it. In that way students can compare their problem-solving strategies. Furthermore, if a student answered a question incorrectly when he took the test, the instructor may ask him to explain the method he employed that led to the wrong answer. Pay special attention to these explanations of errors, since they will show you how *not* to deal with such problems. Learning to recognize and avoid ineffective problem-solving methods is an important part of the training. Also notice the sequences of thoughts used by students who answered the question successfully. Compare the approaches leading to the correct answer with those leading to incorrect answers. Pinpoint how the approaches differ. Most importantly, for every question that you answer incorrectly, be sure you understand exactly why the error occurred, and how you can avoid such an error in the future.

If you are not using this book in a class, have a brother, sister, parent or some other friend take the WASI and then compare your answers and strategies.

Asking people to explain their answers to the test questions, and then to compare the explanations with those of others, accomplishes two things. First, it takes the mystery out of mental tests, making them less threatening should you be required to take such tests in the future. Secondly, research shows that this is an excellent way for people to improve their problem-solving skills. When they work through a test together, explaining and comparing their methods of solution, they learn from each other. They come to recognize ineffective methods, dead ends and pitfalls. They also come to understand how to attack problems effectively and reach correct answers.

Taking the WASI and then discussing it can be a highly valuable learning experience. When your instructor sets aside several class hours for this, use the time and opportunity to your greatest advantage.

WASI TEST
WHIMBEY ANALYTICAL SKILLS INVENTORY

Instructions

This inventory consists of 38 questions. Some of the questions are multiple choice, while others are more complex. For each of the multiple choice questions, circle the answer which you think is correct.

Here are two sample questions. Please try to answer them.

1. If you started with $25.00 and then spent $3.00 to go to a movie, how much would you have left?

   a. $23.00 *b.* $22.00 *c.* $21.00 *d.* $12.00

2. Circle the fifth word in this sentence.

For the first sample question you should have circled alternative *b.*, since $22.00 are left after spending $3.00 for the movie. With the second question you should have circled the word "in," because it is the fifth word in the sentence.

If you have any questions, please ask your instructor to answer them. Otherwise, wait until your instructor asks you to turn the page, then begin.

1. Which word is different from the other 3 words?

   a. yell *b.* talk *c.* pencil *d.* whisper

2. Which letter is as far away from *K* in the alphabet as *J* is from *G*?

   a. *K* *b.* *M* c. *N* *d.* *G* e. *I*

3. If you are facing east and turn left, then make an about-face and turn left again, in which direction are you facing?

   a. east *b.* north c. west *d.* south *e.* southwest

4. Which pair of words fits best in the blanks?

   Arm is to wrist as __________ is to __________.

   a. leg: foot *b.* thigh: ankle c. leg: ankle *d.* leg: knee

5. 20 is to 30 as 10 is to ________?

   a. 5 *b.* 25 c. 60 *d.* 15 *e.* 10

6. Which set of letters is different from the other 3 sets?

   a. *EFGE* *b.* *BCDB* c. *KLML* *d.* *OPQO*

7. In a different language *liro cas* means "red tomato," *dum cas dan* means "big red barn" and *xer dan* means "big horse." What is the word for *barn* in this language?

   a. *dum* *b.* *liro* c. *cas* *d.* *dan* *e.* *xer*

8. Write the 2 letters which should appear next in the series.

C B F E I H L K ___ ___

9. There are 3 separate, equal-size boxes, and inside each box there are 2 separate small boxes, and inside each of the small boxes there are 4 even smaller boxes. How many boxes are there altogether?

a. 24 *b.* 13 *c.* 21 *d.* 33 *e.* some other number

10. Ten full crates of walnuts weigh 410 lb, while an empty crate weighs 10 lb. How much do the walnuts alone weigh?

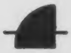
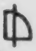
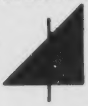
a. 400 lb *b.* 390 lb *c.* 310 lb *d.* 320 lb *e.* 420 lb

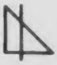
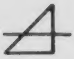
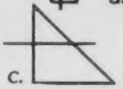
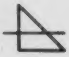
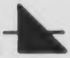
11. One number in the series below is incorrect. What should that number be?

3 4 6 9 13 18 24 33

a. 33 *b.* 7 *c.* 24 *d.* 31 *e.* 32

12. The first figure is related to the second figure in the same way that the third figure is related to one of the answer choices. Pick the answer.

This  is to this  as this  is to?

a.  *b.*  *c.*  *d.*  *e.* 

13. Which pair of words best fits the meaning of the sentence?

__________ the dog was big, he was __________ heavy.

a. Since—not *b.* Although—very

c. Although—not *d.* Because—nevertheless

14. Write the 2 numbers which should appear next in the series.

3 9 5 15 11 33 29 ___ ___

15. An orthopedist is a __________ specialist.

a. brain b. heart c. ear and throat d. lung e. bone

16. An equivocal statement is __________.

a. relevant b. equivalent
c. credible d. somewhat loud
e. ambiguous

17. Three empty cereal boxes weigh 9 ozs and each box holds 11 ozs of cereal. How much do 2 full boxes of cereal weigh together?

a. 20 ozs b. 40 ozs c. 14 ozs d. 28 ozs e. 15 ozs

18. Cross out the letter after the letter in the word pardon which is in the same position in the word as it is in the alphabet.

19. A journey always involves a __________?

a. person b. destination
c. distance d. vehicle
e. preparation

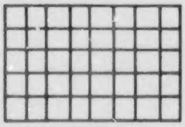
20. In how many days of the week does the third letter of the day's name immediately follow the first letter of the day's name in the alphabet?

a. 1 b. 2 c. 3 d. 4 e. 5

21. Which pair of words is different from the other 3 pairs?

a. walk—slowly *b.* speak—loud
c. read—book *d.* lift—quickly

22. The top 4 figures form a series which changes in a systematic manner according to some rule. Try to discover the rule and choose from among the alternatives the figure which should occur next in the series.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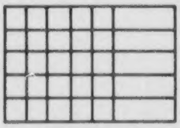
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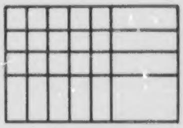
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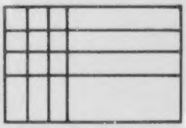
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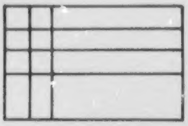
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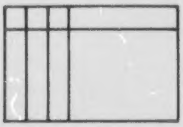
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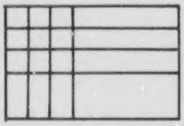
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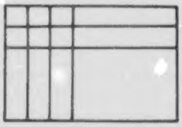
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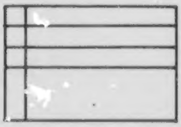

E

23. Which number is repeated first in the following series?

5 9 4 8 2 3 6 1 7 4 7 6 7 8 9 1 5 2 3 5 8 9 5 3 5 4 3 7 1

a. 7 *b.* 8 *c.* 6 *d.* 4 *e.* 5

24. Which pair of words fits best in the blanks?

Oven is to *bake* as __________ is to __________.

a. automobile: carry
b. dishwasher: dishes
c. food: ice
d. vacuum cleaner: rug

25. Write the 3 letters which should come next in this series.

B A A C E E D I I E M M F ___ ___ ___

26. *One-third* is to *9* as *2* is to ________.

a. 6 b. 18 c. 36 d. 54 e. 99

27. *Elephant* is to *small* as ____________ is to ____________.

a. large: little
b. hippopotamus: mouse
c. turtle: slow
d. lion: timid

28. Which word means the opposite of *demise*?

a. nasty b. birth c. accept d. embrace

29. Which set of letters is different from the other 3 sets?

a. *HRTG* b. *NOMP* c. *XACW* d. *LDFK*

30. *Hospital* is to *sickness* as __________ is to __________.

a. patient: disease
b. jail: prisoner
c. doctor: patient
d. school: ignorance
e. nurse: illness

31. A train travels 50 mi when a car travels 40 mi. How many miles will the train travel when the car travels 60 mi?

a. 60 *b.* 50 *c.* 70 *d.* 75 *e.* 80

32. *Heretic* is to *religious* as __________ is to __________.

a. disbelief: faith *b.* adversary: cooperative
c. sinner: punishment *d.* disrespectful: pious

33. How many sixths are in 12/2?

a. 6 *b.* 1 *c.* 36 *d.* 4 *e.* 24

34. *2, 9, 3, 5, 1, 8, 4.* Take the difference between the second [illegible]ber and the next-to-last number, then add it to the fourth number. If this sum is less than 6, write the word "go" in this space __________; otherwise, write the word "stop" in this space __________.

35. Which word is different from the other 3 words?

a. peregrination *b.* pilgrimage
c. outlandish *d.* promenade

36. *3, 6, 4, 2, 5, 9, 1.* Add the second number to the sixth number, then divide by 3 and write the quotient, unless it is greater than 5; in this case add the first number to the next-to-last number and divide by 4. What is your final answer?

a. 3 *b.* 5 *c.* 2 *d.* 4 *e.* some other number

37. Select the answer which is most nearly equivalent in meaning to the following statement.

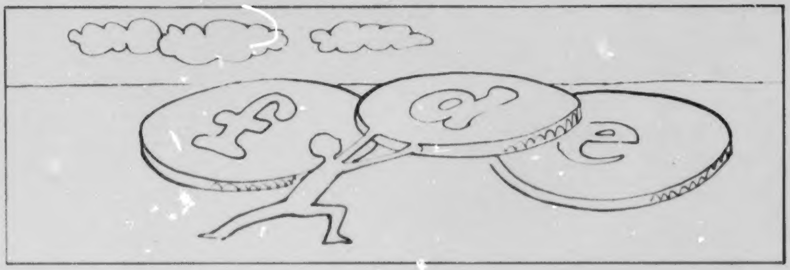
> Show me the man you honor. I know by that symptom, better than any other, what you are yourself.
>
> —Carlyle

a. The works of great scholars should be read and studied.
b. A man can be judged by his works.
c. A man can be judged by those he emulates.
d. Each human being has his own unique worth.

38. *Optimist* is to *pessimist* as ________ is to ________.

a. solace: morose
b. sanguine: morose
c. benefactor: patron
d. eulogy: gloomy

END. When you are finished, check back over your work.



II. ERRORS IN REASONING

One way to improve your analytical skills is to see the types of errors that people frequently make in solving problems, and then guard against those same errors yourself.

Various types of errors undoubtedly came to light in your discussion of the WASI. This chapter analyzes a sample of errors made by students in courses that we have taught. Read through these errors and see how they compare to the ones you made.

Occasionally errors are made on the WASI because people don't have enough information to answer a question. For example, on vocabulary questions (such as question #16) a person might not know the meaning of the words. But most errors are not of this type. Instead, people have sufficient facts yet miss questions because their analyses and reasoning processes break down. Here are four ways in which the breakdowns frequently occur:

1. Person fails to observe and use all the relevant facts of a problem.
2. Person fails to approach the problem in a systematic step-by-step manner. He makes leaps in logic, jumping to conclusions without checking them.
3. Person fails to spell out relationships fully.
4. Person is sloppy and inaccurate in collecting information and carrying out mental activities.

These sources of error tend to be interrelated; however, one may be more prominent than the others with some particular person or problem. You will see examples of all four sources of error below.

Error Analysis of Sample IQ Questions

In this section we will review errors made on the following questions from the WASI: Questions 7, 9, 10, 12, 18, 22, 27 and 28. These questions are representative of the types of problems that are found on most IQ tests. Let's begin with question 7.

Question 7

In a different language *liro cas* means "red tomato," *dum cas dan* means "big red barn" and *xer dan* means "big horse." What is the word for *barn* in this language?

a. dum *b. liro* *c. cas* *d. dan* *e. xer*

This is a fairly easy question. Yet errors are made when people don't systematically compare phrases and sequentially match words until they isolate the word for barn. Instead of working step-by-step through all the information, people with poor thinking habits just make a cursory examination of the phrases and then try to approximate the answer, often choosing alternatives *c, d,* or *e.*

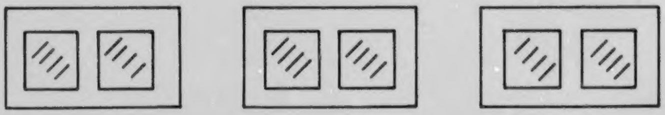
Question 9

There are 3 separate, equal-size boxes, and inside each box there are 2 separate small boxes, and inside each of the small boxes there are 4 even smaller boxes. How many boxes are there altogether?

a. 24 *b.* 13 *c.* 21 *d.* 33 *e.* some other number

This question is especially interesting because its solution is quite straightforward, yet it is often missed. It illustrates the lack of skill which some people have in spelling out ideas fully and accurately.

The simplest way to solve this problem is to draw a diagram representing correctly the relationship of the boxes.



A student who chose answer *c* described her inadequate reasoning as follows:

> I pictured the three boxes and the two smaller boxes inside the three boxes . . . I added three plus two (which gave five) and counted the four other boxes twice. Five plus eight gave me 13.

This student didn't spell the diagram out fully. Instead she went ahead and added numbers without carefully considering exactly which numbers should be added and why.

Many people approach mathematics problems in this way. They perform inappropriate numerical operations because they don't clarify in their own minds the exact relationship of the facts in the problem.

Question 10

Ten full crates of walnuts weight 410 lb, while an empty crate weighs 10 lb. How much do the walnuts alone weigh?

a. 400 lb *b.* 390 lb *c.* 310 lb *d.* 320 lb *e.* 420 lb

This is a conventional math word problem, something which frightens a large percentage of students and adults. They feel that a special inborn ability is required for mathematics, an ability in which they rate a great big zero.

When you look at this problem closely, you see that it doesn't require any mysterious ability. All that this problem demands is that the facts be spelled out fully and accurately. Once that is done, the remaining arithmetic is simple.

Here is a diagram which spells the facts out fully. It shows that the total weight is composed of 11 parts, the weight of the 10 crates and the weight of the walnuts.

Total Weight: 410 Pounds

<table>
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lb</td></tr>
<tr><td colspan="10">Walnut Weight: 310 pounds</td></tr>
</table>

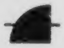
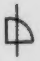
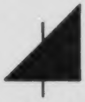
You may not have actually visualized this diagram in working the problem. But conceptually you used a similar model. You had to sort the total weight into the various parts shown in the diagram in order to compute the answer.

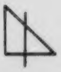
Students who have trouble with this and other math problems haven't learned to spell numerical relationships out fully so that correct calculations can be made. For example, a mistake frequently made with this problem is to answer 400 pounds, showing that the person did not take all 10 boxes into account.

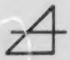
A revealing answer to this problem is alternative *e*, which is 420 pounds. This answer is sometimes selected, even though it doesn't make sense. The walnuts alone can't possibly weigh more than the walnuts plus the crates. This answer shows how anxious and flustered some people get in doing math. They see two numbers in the problem, 410 pounds and 10 pounds, so they quickly add the two numbers together, giving less detailed thought to the meaning of the problem than they give to ordering lunch. They feel conquered by math, smartly defeated, and so they never even begin calling their analytical skills out to battle.

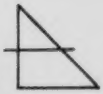
Question 12

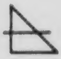
The first figure is related to the second figure in the same way that the third figure is related to one of the answer choices. Pick the answer.

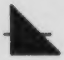
This  is to this  as this  is to?

a. 

b. 

c. 

d. 

e. 

This is a fairly easy figural problem. Still, people make errors because they are inaccurate in observing or using the given information in understanding the analogy and selecting an answer. For example, they may neglect to change the shading and choose answer *e*. Or they may neglect size and choose *c*.

Question 18

Cross out the letter after the letter in the word pardon which is in the same position in the word as it is in the alphabet.

This is a fairly difficult verbal reasoning problem. One interesting thing about it is that while on the surface it appears to be very different from question 12 (above), errors on the two questions come about for the very same reasons. People fail to search out and use all the available information.

One frequent error is to cross out the *d* in *pardon,* rather than crossing the letter after the *d.* The person making this error has lost part of the problem.

Another less frequent error is to cross out the *d* in *word,* as shown below.

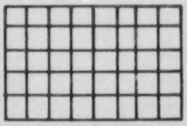
> Cross out the letter after the letter in the word pardon which is in the same position in the wor~~d~~ as it is in the alphabet.

People who make this error haven't learned to work step-by-step through a complex sentence. They don't think through the sentence in the following way:

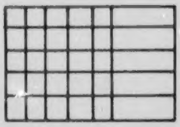
> Cross out the letter after the letter (so I have to cross out a letter) in the word (cross out a letter in some word) pardon (so pardon must be the word).

Question 22

The top four figures form a series which changes in a systematic manner according to some rule. Try to discover the rule and choose from among the alternatives the figure which should occur next in the series.



1



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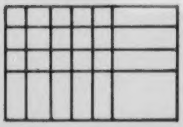

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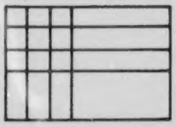

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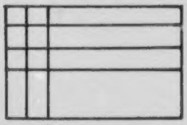
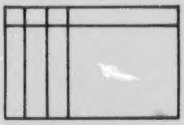
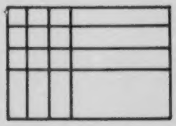


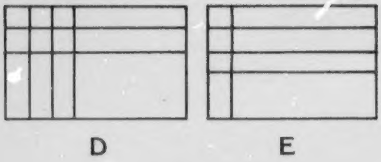
A



B



C



Here is another problem which is missed frequently although it involves no obscure or esoteric ideas. Errors arise from inaccuracies in making observations and building them into a system or rule which leads to the answer. For example, a student who chose answer *e*, explained his thinking as follows:

> I noticed that first there are some lines taken away. Then there are more lines taken away going the other way. Then there are more lines taken away going up and down. So I guess the answer should take more lines away. I guess answer *e*.

I asked him whether he had counted the lines to see exactly how many were deleted in each figure and he answered that he had not, it seemed too confusing. As you see, this student had never learned to accurately keep track of facts in problem solving.

Question 27

Elephant is to *small* as ______________ is to ______________ .

a. large: little	*b.* hippopotamus: mouse
c. turtle: slow	*d.* lion: timid

Verbal analogies like this one play a large role on tests such as the SAT, GRE, and various other aptitude tests and IQ measures. Analogy questions are widely used because they tap a person's ability to define ideas and relationships fully and accurately.

With this particular question, students whose leanings are nonanalytical read "elephant is to small" and quickly decide on "large: little" as the answer. Such a student doesn't spell out in his mind that an "elephant" is an animal and "small" is a quality, whereas "large" and "little" are both qualities. He reaches conclusions on the basis of quick immediate impressions rather than thorough, step-by-step interpretations, and as a result he often misses significant dimensions of relationships.

Question 28

Which word means the opposite of *demise?*

a. hasty *b.* birth *c.* accept *d.* embrace

This is a vocabulary question, and there is hardly a mental test on the market which doesn't include at least a few of them. In this question you are given a word and asked to find another word which is opposite in meaning. At other times (as with question 16) you must find the word that is similar in meaning.

You may have noticed that vocabulary questions are different than the other test questions we have looked at. Most test questions require reasoning and problem solving, whereas vocabulary questions are mainly a matter of recall.

This apparent disparity is cleared up when you consider how vocabulary is acquired. A rich vocabulary is the by-product of careful thinking in verbal communication. An analytical thinker listens and reads for complete understanding of relevant ideas. When he encounters a new word he tries to estimate its meaning from the context. If he's still uncertain, he takes out the dictionary and studies the various entries, interpreting and contrasting ideas and terms until he is sure of the word's definition. In short, although vocabulary questions do not require problem solving at the time a person takes a test, these questions do reflect the precise thinking that the person employed in acquiring his vocabulary.

Summary

In this chapter we have looked at the types of test questions that are used to measure reasoning ability, and have seen that errors are primarily caused by a lack of accuracy and thoroughness in thinking. Research has shown that accuracy and thoroughness are mental habits which can be cultivated through training and exercise. This book provides some of that training. But you need to go further on your own. With everything you read, practice carefulness in comprehending ideas and relationships. And in solving problems, continually check yourself for accuracy and completeness. Initially this may be difficult and require conscious discipline, just as learning good typing habits or correct swimming movements may at first be difficult. Gradually, however, the attitudes and skills of tight reasoning will become as natural to you as swimming, skating, driving, typing, or any of the various other skills that you have learned with practice and time.

CHECKLIST FOR PROBLEM SOLVING

Following is a checklist of sources and types of errors in problem solving. Some of the items overlap, referring to different aspects of the same fault in working problems, but this overlap is unavoidable because the various factors that underlie problem-solving skill are interrelated. Read the checklist aloud, discussing any items that are unclear. Then, as you solve problems, be careful not to make these errors. If you recognize some particular error to which you are especially prone, take extra pains to guard against it. Also, when you are listening to another student solve a problem, watch his approach for errors of the type listed below.

Inaccuracy in Reading

1. Student read the material without concentrating strongly on its meaning. He (she) was not careful about whether he understood it fully. He read sections without realizing that his understanding was vague. He did not constantly ask himself: "Do I understand that completely?" This showed up in errors he made later.

2. Student read the material too rapidly, at the expense of full comprehension.

3. Student missed one or more words (or misread one or more words) because the material was not read carefully enough.

4. Student missed or lost one or more facts or ideas because the material was not read carefully enough.

5. Student did not spend enough time rereading a difficult section to clarify its meaning completely.

Inaccuracy in Thinking

6. Student did not constantly place a high premium on accuracy—He did not place accuracy above all other considerations such as speed or ease of obtaining an answer.

7. Student was not sufficiently careful in performing some operation (such as counting letters) or observing some fact (such as which of several figures is the tallest).

8. Student was not consistent in the way he interpreted words or performed operations.

9. Student was uncertain about the correctness of some answer or conclusion, but did not check it.

10. Student was uncertain about whether a formula or procedure he used to solve the problem was really appropriate, but did not check it.

11. Student worked too rapidly, which produced errors.

12. Student was inaccurate in visualizing a description or a relationship described in the text.

13. Student drew a conclusion in the middle of the problem without sufficient thought.

Weakness in Problem Analysis; Inactiveness

14. Student did not break a complex problem into parts. He did not begin with a part of the problem that he could handle in order to get a foothold. He did not proceed from one small step to the next small step, being extremely accurate with each one. He did not use the parts of the material he could understand to help him figure out the more difficult parts. He did not clarify his thoughts on the parts he did understand and then work from there.

15. Student did not draw upon prior knowledge and experience in trying to make sense of ideas which were unclear. He did not try to relate the written text to real, concrete events in making the meaning clear and understandable.

16. Student skipped unfamiliar words or phrases, or was satisfied with only a vague understanding of them, rather than trying to obtain a good understanding from the context and the remainder of the material.

17. Student did not translate an unclear word or phrase into his own words.

18. Student did not use the dictionary when necessary.

19. Student did not actively construct (mentally or on paper) a representation of ideas described in the text, where such a representation could have helped in understanding the material.

20. Student did not evaluate a solution or interpretation in terms of its reasonableness, i.e. in terms of his prior knowledge about the topic.

Lack of Perseverance

21. Student made little attempt to solve the problem through reasoning because he lacked confidence in his ability to deal with this type of problem. He took the attitude that reasoning would not work with this problem. He felt confused by the problem, so didn't start systematically by clarifying the portions of the problem which were readily understandable, and then attempting to work from there.

22. Student chose an answer based on only a superficial consideration of the problem—on an impression or feeling about what might be correct. Student made only a superficial attempt to reason the problem, then guessed an answer.

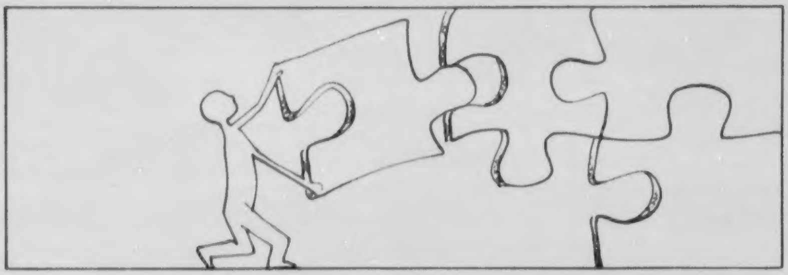
23. Student solved the problem in a mechanical manner, without very much thought.

24. Student reasoned the problem part way through, then gave up and jumped to a conclusion.

Failure to Think Aloud

The items above apply to all academic problem solving. The last item refers specifically to the procedure used in this course.

25. Student did not vocalize his thinking in sufficient detail as he worked through the problem. At places he stopped and thought without vocalizing his thoughts. He performed a numerical computation or drew a conclusion without vocalizing or explaining the steps he took.



III. PROBLEM-SOLVING METHODS

Introduction

If you are using this book in a class your teacher may ask you to work in pairs as you solve the problems. One partner should read and think aloud, while the other partner listens. On subsequent problems the partners should change roles, taking turns as problem solver and listener.

You can also use this procedure if you are not in a class, but are working through the book with another person.

Some people find reading and thinking aloud a little awkward at first, but thousands of people have already used this book and have found they adjust to the procedure quickly. Here is the reason that you are asked to read and think aloud.

Thinking Is A Hidden Skill

The ability to analyze complex material and solve problems is a skill—just like any other skill such as the ability to play golf or the ability to drive an automobile. However, there is a peculiar difficulty involved in teaching analytical skill. Generally there are two phases to teaching a skill. First the skill is demonstrated to the student. Then he is guided and corrected as he practices it. For example, golf is taught by showing the beginner how to grasp the club, how to place his feet, how to move his arms and his body as he swings. The beginner can watch a golf pro—he can even watch a slow motion film of the pro in action—and in this way can learn the pro's technique. Furthermore, the pro can observe the beginner as he practices, he can point out his flaws, and he can show him how to improve.

In contrast to playing golf, analyzing complex material is an activity which is generally done inside your head. This makes it somewhat difficult for a teacher to teach and for a learner to learn. In other words, a beginner cannot observe how an expert thinks and solves problems. And the expert has trouble demonstrating his technique to a beginning student. There is one way to reduce this difficulty—have people think aloud while they solve problems. If both students and experts vocalize their thoughts as they work through complex ideas and relationships, the steps that they take are open to view and their activities can be observed and communicated.

In this book, the procedure of asking people to think aloud while they solve problems is applied in two ways. Experienced problem solvers (a group of graduate students and professors) were asked to think aloud as they solved the problems that are presented in the book. Their responses were tape-recorded, and then the steps they took in solving a problem were summarized and written out. These summaries are presented under the heading Problem Solution. In other words, the problem solution which follows each problem is a summary of steps taken by an experienced problem solver as he or she worked the problem aloud.

The second application of the procedure consists in asking you, the reader, to think aloud as you work each of the problems. In doing this, you make your thinking visible to other people so that they can observe your attack on a problem. Thus, they can learn the techniques you use; they can help point out any errors you make, and they can compare the steps you take with the steps listed in the problem solution. Furthermore, you will find that by thinking aloud you will be able to look at your own thinking activities more carefully. You will be able to see exactly what strategies you use, and what difficulties you have in solving a problem.

Research has shown that this is an effective way for students to improve their problem-solving skills: work together, think aloud, learn from each other, and read how experienced problem solvers approached the same problems.

Quiz Yourself

1. What are the two phases of teaching a skill such as golf or swimming?
2. What special barrier is met in trying to teach analytical reasoning skill?
3. How has this program attempted to overcome the difficulty encountered in teaching analytical reasoning? How has it handled the two phases of teaching the skill?

Thinking Aloud

In this book you are asked to do your thinking aloud. Naturally you cannot vocalize all of the mental processing that you do. For example, you cannot explain how you know the meaning of every word you read in a problem. However, when you are unsure about a word or an idea, and you have to stop and think about it, do this thinking aloud. As a rule of thumb, try to think aloud as much as possible while doing the exercises. Spelling out your thoughts—especially at sections of a problem which you find difficult or confusing—is the safest way to insure that you do not skip steps in your reasoning, nor miss facts in drawing conclusions. In other words, you will find that vocalizing your thoughts forces you to be more careful and thorough in analyzing ideas.

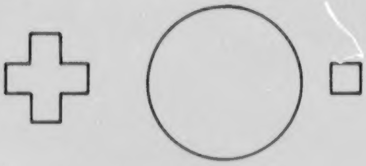
Thinking aloud while solving problems requires a certain amount of practice. At first you may find it a little difficult to vocalize your thoughts as you work a problem—to express in words the steps you take in solving the problem. However, research studies show that most students get used to this procedure quickly.

In order to illustrate the procedure of thinking aloud, the response of an experienced problem solver who was asked to think aloud as she worked a problem is reproduced on the following pages.* As you read this protocol, try to follow all of the steps and activities of the problem solver.

Problem Solver's Response

Original Problem

If the circle below is taller than the square and the cross is shorter than the square, put a *K* in the circle. However, if this is not the case, put a *T* in the second tallest figure.



Problem Solver's Response

Note: Read both the comments on the left and the problem solver's report on the right. Quotation marks show when the problem solver read aloud. The absence of quotation marks indicates that the problem solver was thinking aloud.

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*The problem solver is an outstanding medical student who earned a Master's degree in Comparative Literature before deciding to enter the field of medicine.

The Problem Solver read and thought aloud.

"If the circle below is taller than the square and the cross is shorter than the square, put a *K* in the circle."

Let me start again.

"If the circle below" . . . I'll put my finger on the circle . . . "is taller than the square" . . . Yes, the circle is taller than the square.

"And the cross is shorter than the square" . . . I'll move my finger from the cross to the square and compare them . . . this part is false . . . the cross is not shorter than the square.

"Put a *K* in the circle."

So I shouldn't put a *K* in the circle. Part of the statement is false. I would only write *K* if both the first part and the second part were true.

I should read the whole sentence again and see if my conclusion is correct.

"If the circle is taller than the square" . . . Yes . . . "and the cross is shorter than the square" . . . No . . . "Put a *K* in the circle" . . . I didn't. That's correct.

As the Problem Solver reread the sentence, she moved her finger first from the circle to the square and then from the cross to the square.

I'll continue to the next part of the problem.

"However, if this is not the case" . . . and it isn't the case . . . "put a *T* in the second tallest figure."

The second tallest figure is the cross so I'll put a *T* in the cross.

Another Example

Here is the response that an experienced problem solver (a lawyer) gave to another problem. Note how carefully he checks everything, and even stops for a moment in deciding right from left.

Original Problem

	Yes	No

If the word *sentenⓒe* contains less than 9 letters and more than 3 vowels, circle the first vowel. Otherwise circle the consonant which is farthest to the right in the word.

Problem Solver's Response

The Problem Solver read the entire problem aloud.	"If the word *sentence* contains less than 9 letters and more than 3 vowels, circle the first vowel. Otherwise circle the consonant which is farthest to the right in the word."
	I'll start from the beginning.
	"If the word *sentence* contains less than 9 letters."
The Problem Solver pointed to the letters with his pen as he counted.	I'll count the letters in *sentence. 1, 2, 3, 4, 5, 6, 7, 8.* Let me check it. *1, 2, 3, 4, 5, 6, 7, 8.* So it does have less than 9 letters. I'll write the word *yes* above the problem. That way I'll remember it.
The Problem Solver wrote yes over the sentence (see original problem).	
The Problem Solver resumed reading.	"And more than 3 vowels"
The Problem Solver pointed with his pen as he counted.	*1, 2, 3.* Let me check that. *1, 2, 3.* It contains exactly 3 vowels, not more than 3 vowels. I'll write *no* on the problem to remind me.
	"Circle the first vowel."
	So I won't do that.
	"Otherwise circle the consonant which is farthest to the right in the word."
	The consonant farthest to the right? Let me see. Which is my right hand? This is my right hand. OK, so the last letter is the one farthest to the right. But the last letter is *E.* The next letter over is *C.* So it is the consonant farthest to the right. I'll circle the *C.*

Methods of Good Problem Solvers

The Problem Solvers' Responses that you just read illustrate several characteristics of good problem solvers. These characteristics have been studied by researchers and they will be summarized here in five sections.

1. *Positive Attitude*

First of all, good problem solvers have a strong belief that academic reasoning problems can be solved through careful, persistent analysis. Poor problem solvers, by contrast, frequently express the opinion that "either you know the answer to a problem or you don't know it, and if you don't know it you might as well give up or guess." Poor problem solvers haven't learned that a problem may at first appear confusing—that the way to work the problem may not at first be obvious—but that through carefully breaking the problem down, by pinpointing first one piece of information and then another, a difficult problem can be gradually analyzed. Poor problem solvers lack both confidence and experience in dealing with problems through gradual (sometimes lengthy) analysis.

2. *Concern for Accuracy*

Good problem solvers take great care to understand the facts and relationships in a problem fully and accurately. They are almost compulsive in checking whether their understanding of a problem is correct and complete. By contrast, poor problem solvers generally lack such an intense concern about understanding. For example, good problem solvers sometimes reread a problem several times until they are sure they understand it. Poor problem solvers, on the other hand, frequently miss a problem because they do not know exactly what it states. Quite often they could have found out if they had been more careful, if they had reexamined and thought about the problem analytically. But poor problem solvers have not learned how important it is to try to be completely accurate in understanding all of the ideas of a problem. (Recall how the experienced problem solvers in the last two exercises reread sections of the problem to be sure they understood them fully, and rechecked even their simplest calculations.)

3. *Breaking the Problem into Parts*

Good problem solvers have learned that analyzing complex problems and ideas consists of breaking the ideas into smaller steps. They have learned to attack a problem by starting at a point where they can make some sense of it, and then proceeding from there. In contrast, poor problem solvers have not learned the approach of breaking a complex problem into subproblems—dealing first with one step and then another. In the problems which follow, you will see many examples of how complex problems can be worked one step at a time.

4. *Avoiding Guessing*

Poor problem solvers tend to jump to conclusions and guess answers without going through all the steps needed to make sure that the answers are accurate. Sometimes they make intuitive judgments in the middle of a problem without checking to see whether the judgments are correct. At other times they work a problem part of the way, but then give up on reasoning and guess on an answer. Good problem solvers tend to work problems from beginning to end in small, careful steps.

The tendency for poor problem solvers to make more errors—to work too hastily and sometimes skip steps—can be traced to the three characteristics already discussed. First, poor problem solvers do not strongly believe that persistent analysis is an effective way (in fact the only way) to deal with academic reasoning problems. Thus their motivation to persist in working an entire problem precisely and thoroughly—until it is completely solved—is weak.

Second, poor problem solvers tend to be careless in their reasoning. They have not developed the habit of continuously focussing and checking on the accuracy of their conclusions. And third, they have not learned to break a problem into parts and work it step-by-step. As a result of these three characteristics, poor problem solvers have a strong tendency to make hasty responses as they work academic reasoning problems, causing errors in both simple computations and in logic.

5. *Activeness in Problem Solving*

The final characteristic of good problem solvers is the tendency to be more active than poor problem solvers when dealing with academic reasoning problems. Put simply, they do more things as they try to understand and answer difficult questions. For example, if a written description is hard to follow, a good problem solver may try to create a mental picture of the ideas in order to "see" the situation better. If a presentation is lengthy, confusing, or vague, he will try to pin it down in terms of familiar experiences and concrete examples. Furthermore, he will ask himself questions about a problem, answer the questions, and "talk to himself" as he clarifies his thoughts. He may also count on his fingers, point to things with his pen, write on the problem, make diagrams or use other physical aids to thinking. All in all, good problem solvers are active in many ways which improve their accuracy and help them get a clearer understanding of ideas and problems.

Quiz Yourself

The text listed five areas in which good academic problem solvers differ from poor academic problem solvers. These are: *1.* motivation and attitude toward problem solving; *2.* concern for accuracy; *3.* breaking problems into parts; *4.* guessing; and *5.* activeness in problem solving. Describe and explain three of these areas in detail.

Role of the Listener When Working With A Partner

As noted earlier, if you are using this book in a class your teacher may ask you to work in pairs. One partner should read and think aloud, while the other partner listens. On subsequent problems the partners should change roles, taking turns as problem solver and listener.

The partner who listens plays an important role in the learning process. He should not sit back inattentively with his mind elsewhere. Instead, he should concentrate on two functions. He should: *1.* continually check accuracy, and *2.* demand constant vocalization.

1. *Continually Check Accuracy*

Since accuracy is all-important, the listener should continually check the accuracy of the problem solver. This includes every computation he makes, every diagram he draws, every conclusion he reaches. In other words, the problem solver's accuracy should be checked at every step of the problem, not just when he gives his final answer. For example, if in working the problem shown earlier the problem solver concluded that the word *sentence* has nine letters, the listener should have immediately caught the error and pointed it out.

Catching errors involves several activities. First, the listener must actively work along with the problem solver. He should follow every step the problem solver takes, and he should be sure he understands each step. If the listener takes a passive attitude—if he does not actively think through each step—he won't know for sure whether or not the problem solver's steps are totally correct.

Second, the listener should never let the problem solver get ahead of him. This may often mean that the listener will have to ask the problem solver to wait a moment so that he can check a conclusion. In this program the emphasis is on accuracy, not on speed. Both the problem solver and the listener should concentrate on accuracy. If the listener needs a moment to verify a conclusion, this will give the problem solver a chance to go over his work and check his own thinking. The problem solver should, in the back of his mind, constantly have the thought "Is that correct—should I check that?" as he works a problem. This will slow him down a little so that the listener will be able to keep pace. However, if the problem solver is working too hastily, at the expense of accuracy, the listener should ask him to slow down—so that he can follow accurately and analytically. Moreover, even if the problem solver is not working too hastily to be accurate, the listener may still occasionally ask him to stop a moment while he checks a point he is unsure of.

Third, the listener should not work the problem separately from the problem solver. When some listeners first learn the procedure used in this program, they turn away from the problem solver and work the problem completely on their own. Occasionally they even finish the problem long before the problem solver. This is incorrect. The listener should listen. He should actively work along with the problem solver, not independently of him.

Finally, when the listener catches an error he should only point it out —**he should never give the correct answer.** By the same token, if the listener sees an answer or a conclusion before the problem solver sees it, he should not furnish it, but should wait for the problem solver to work it out. If the problem solver seems completely stuck, the listener may provide a suggestion on the first step to take. But he should not actually take the first step and obtain a partial answer. Instead, the problem solver should do all the work.

In summary, the listener should understand that he is not being picky or overly critical of his partner when he points out errors. He is helping him improve his scholastic problem solving skill—a skill which will be useful in all academic courses. The listener should check every step taken and every conclusion reached by the problem solver. He should never let the problem solver go on to a second step until he checks the first one. And when he detects an error he should point it out without actually correcting it.

2. *Demand Constant Vocalization*

The second function of the listener is to insure that the problem solver vocalizes all of the major steps he takes in solving a problem. Thinking aloud is a primary part of this program. It is the only way to communicate and to monitor thinking. It should not be neglected. Even the solution of simple problems should be vocalized entirely—so that vocalizing can be done easily when difficult problems are met. If the problem solver skips through one or more steps without thinking aloud, the listener should ask him to explain his thoughts at that point.

An Example

The roles of the problem solver and listener are demonstrated in the following dialogue between two graduate students working a problem. As you read through the example, notice how the listener helps the problem solver both to be more accurate and to keep talking. The listener is always following what the problem solver is saying and at the same time looking for ways to make the problem solver think harder about the problem.

The Problem Solver begins by reading the problem twice

Sally < Bill < Judy

Problem Solver

"'Bill, Judy and Sally have the occupations of teacher, plumber and teamster but not necessarily in that order. Bill is shorter than Judy but taller than Sally. The plumber is the tallest and the teamster is the shortest. What is Judy's occupation?' Now you want to see how to solve that? OK. Let's see. I would read it again. 'Bill, Judy and Sally have the occupations of teacher, plumber and teamster but not necessarily in that order. Bill is shorter than Judy but taller than Sally. The plumber is the tallest and the teamster is the shortest. What is Judy's occupation?' So I would go back to where it says Bill is shorter than Judy but taller than Sally and I would note that in some way. Um, I guess using lines to indicate, you know, some kind of length. No I wouldn't. I would just write their names down in the order that it gives. So I'd have Bill is less than Judy but taller than Sally—common inequality."

Listener

"OK. So you're using inequality signs here?"

Here the Listener probably understands the Problem Solver's notation but is looking for a more detailed explanation. It is always a good idea to ask the Listener to explain any new symbol.

teamster __ plumber

Problem Solver

"Yeah. Just to show and er, well they indicate height; unequal height. So it could be read the height of Bill is less than the height of Judy and greater than the height of Sally. Then it says the plumber is the tallest of these three so I would write down the word plumber and the teamster is the shortest. And so I where I'd put whatever occupation er, in the middle indicating that the person who is occupied in that manner has the middle height. And then to the left of that I'm going to write teamster."

Listener

"Ok. So first you wrote plumber and you put that on the right-hand side?"

The Listener is trying to get the Problem Solver to explain the diagram in more detail.

Problem Solver

"Right. And then I left a space for some other occupation and then I wrote teamster on the left-hand side, indicating that the teamster is the shortest and the plumber is the tallest."

Listener

"Why do you put the plumber on the right-hand side? Any reason for that?"

Note that while this may seem like a silly question it does get the Problem Solver to reflect more carefully about the problem. In particular, it forces an explicit statement of the implied connection between the two diagrams.

Problem Solver

"No, I could have just as easily put the plumber on the left-hand side except that above it I have the inequality going from right to left meaning that Judy who is the tallest and then Bill who is the next tallest, and then Sally who is the next tallest. So that's probably why I have plumber who is the tallest on the right-hand side under Judy, and teamster who is the shortest on the left-hand side under Sally."

Listener

"Do you think that's why you put it there?"

Problem Solver

"Yeah, I think that's why I put it there. Because I had set the precedent with Sally, less than Bill, less than Judy, going from left to right.

"Uh, so the final question is what is Judy's occupation, and I think that Judy is the plumber." (Pause)

Listener

Whenever the Problem Solver is quiet for more than a few seconds the Listener should ask for verbalization.

"What are you thinking now?"

Problem Solver

"That Judy is the plumber. That Judy is the plumber and I'm going to write um, teacher in between teamster and plumber."

Listener

Even though the Problem Solver seems to have solved the problem, the listener keeps asking questions to make sure the problem is fully understood.

"Why?"

Problem Solver

teamster, teacher, plumber

"Because I think that uh, the teacher is of medium height. But I'm not sure so I'm going to go and check everything now. Bill, Judy and Sally have the occupations of teacher, plumber and teamster but not necessarily in that order, meaning that Bill's not the teacher and Judy's not the plumber and Sally's not the teamster."

Listener

Here the Listener notices that the Problem Solver's words suggest confusion between the terms "not necessarily" and "not." A simple question prompts the Problem Solver to be more careful.

"It means that they're not that?"

Problem Solver

"It says 'but not necessarily in that order,' yeah so Bill does not have to be the teacher. I think when you say 'that order' you mean respectively. You're saying not necessarily respectively. So Bill is not necessarily the teacher."

Listener

"Uh-huh."

Problem Solver

"Judy's not necessarily the plumber and Sally's not necessarily the teamster but it could be that way." (Pause)

Listener

Here again the Listener prompts the Problem Solver to keep talking.

"So what are you thinking now?"

Problem Solver

"So I was thinking that this inequality that I have written down is correct because it says that Bill is shorter than Judy and taller than Sally. So if Bill is shorter than Judy and taller than Sally, then Judy must also be taller than Sally because Bill is shorter than Judy, and Bill is taller than Sally. So if Judy is taller than Bill then she must be taller than Sally. So I'm fine with that particular statement. The plumber is the tallest and the teamster is the shortest. And since Judy is taller than Bill, and Bill is taller than Sally and then Judy is taller than Sally so Judy is the tallest and the plumber is the tallest and so Judy is the plumber. And the teamster is the shortest and since Sally is let's see—(mumbles to self)—So Judy is shorter than Bill and Judy is shorter than Sally and the teamster is the shortest so I think that Judy is the teamster."

Listener

The Listener tries to see if the Problem Solver notices that this contradicts the earlier statement.

"So you just said Judy is the teamster."

Problem Solver

"Right. What did I say before, then, she was the plumber?"

Listener

"Uh-huh."

Problem Solver

"Did I say that?"

Listener

"Yeah."

Problem Solver

"And the teamster is the shortest. No, Sally's the teamster."

Listener

"So what were you just looking at?"

Problem Solver

"Just, you know, momentary dyslexia. Yeah. I'm going to say that Sally's the teamster, and Bill's the teacher and Judy's the plumber."

Listener

"Are you sure?"

The Listener should always check to be sure the Problem Solver is confident before the pair move on to the next problem.

The previous example involved two skilled students. It is likely that the first time you work a problem the Problem Solver will have a harder time talking. The following example illustrates how the Listener can help get a stubborn Problem Solver to talk more.

Problem

If the second letter in the word *west* comes after the fourth letter in the alphabet, circle the letter A below. If it does not, circle the B.

A **B**

Problem Solver reads the problem.

Problem Solver

"'If the second letter in the word *west* comes after the fourth letter in alphabet, circle the letter A below. If it does not, circle the B.'"

Listener

"You said 'in alphabet' not 'in *the* alphabet.'"

Problem Solver

"Oh yeah." (Pause)

Listener

"What are you thinking?"

Problem Solver

"Nothing. I am just looking at it." (Pause)

Listener

"What are you looking at?"

Problem Solver

"It's A. I circle the A."

Listener

"Wait a minute. You just said you weren't thinking. Now you say it's A. How did you get that?"

Problem Solver

"Well, the fourth letter is D."

Listener

"Yes."

Problem Solver

"And so I circle the A."

Listener

"How do you get the fourth letter is D?"

Problem Solver

"A, B, C, D. I count."

Listener

"OK. So how come you circle A?"

Problem Solver

"Because that's what it says to do."

Listener

Here the Listener lies deliberately.

"I think you are wrong."

Problem Solver

"I am wrong?" (Pause) "You mean it's B. It can't be B because the letter E comes after the letter D."

Listener

"Yeah?"

Problem Solver

"And it says that if the second letter in west, which is e, comes after the fourth letter in the alphabet circle the A. So I did."

The Listener tests the Problem Solver's confidence by trying another interpretation.

Listener

"Yes, but e comes before h, which is the fourth letter in alphabet."

Problem Solver

"Hmm. You think they want the fourth letter in *alphabet*? (Pause) No they don't, they said *the* alphabet not in alphabet. You pointed that out to me earlier."

Listener

"Oh."

Problem Solver

"So it's A."

Listener

"Wait, tell me all over why you think it is A—go slowly."

Problem Solver

"Well the problem asks you to circle the A if the second letter in *west* which is *e* comes after the fourth letter in the alphabet which is *d*. And *e* does come after *d* so I circle the A."

Listener

"Are you sure?"

Problem Solver

"Yes."

In this example the Listener never gets the Problem Solver to talk in adequate detail, but through a variety of techniques does get the Problem Solver to talk some. With additional practice both students should eventually be able to reach the level of skill demonstrated in the first example.

Summary

As you work problems together, you will see places where the problem solver failed to vocalize his thoughts, and you will see errors occurring because of hastiness and failure to recheck. The problem solver may neglect to concentrate fully enough on accuracy, or he may forget to approach problems in a systematic, step-by-step manner. Sometimes the errors will be minor, sometimes they will set the problem solution on an entirely wrong track. With experience, you should become sensitive to these types of errors and catch them quickly.

Quiz Yourself

1. What are the two roles of the listener?

2. Describe the activities of the listener in checking for the problem solver's accuracy. According to the text, what are two things the listener should not do?

Problem Solutions

Many of the problems in this book are followed by a section labeled Problem Solution. The problem solution breaks the problem into a series of steps and shows how the problem can be solved in a step-by-step manner. The steps were obtained by asking a good problem solver to think aloud as he worked the problem and then summarizing his steps so they would be easy to read.

After you and your partner agree on the answer to a problem, turn to the problem solution to check that the answer is correct. The steps which are listed may not be identical to the way in which you solved the problem, but the final answer should be the same. If you had any difficulty with the problem, review all the steps in the solution to see whether they differ from the approach you used.

Your instructor may give quizzes in problem solving, asking you to solve problems similar to the ones in the book. Therefore, it will be worth your while to look over the problem solutions carefully to insure you can solve all the problems effectively.

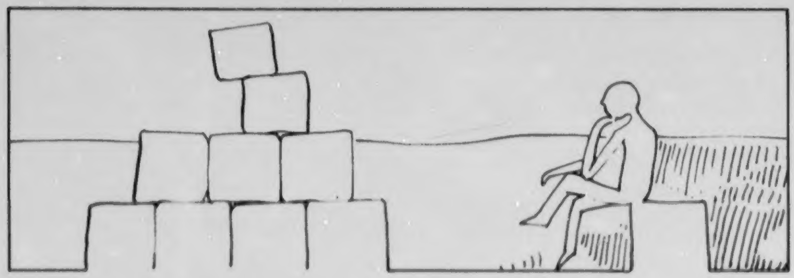
Devising Problems

For homework and additional classwork, your instructor will ask you to devise problems similar to certain of the ones in this chapter. Devising problems based on other problems allows you to see them from the inside out. You come to understand how the various parts of the problems operate and how they relate to each other. Don't be surprised to find that devising good problems can require much more thinking than solving them. Just take your time changing the various problem elements until you are totally satisfied.

Devise each problem on your own. Then have another student solve it, doing his reading and thinking aloud, and also writing out the steps in his solution. Check to be sure he doesn't skip any steps. His written solution should be as complete as the solutions you read in earlier sections. Writing out complete solutions is a powerful exercise for strengthening problem-solving skills as well as writing skills.

Place just one problem per page so that the solution can be written below it. Make the problems difficult enough to be challenging. But be certain they are logically sound and solvable before asking other students to work them.

After your problems have been solved, turn them in to your instructor.



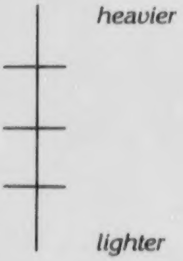
IV. VERBAL REASONING PROBLEMS

Introduction

In this chapter each problem is followed by a problem solution. Scan the solution to see if your answer is correct. If it is not correct, read the entire problem solution aloud. As you read the solution, notice how the problem is analyzed into steps. Also notice any diagrams or other problem-solving aids and techniques that are employed. Make use of these techniques whenever they are appropriate in solving later problems.

Problem 1

Tom is heavier than Fred but lighter than Marty. Write the names of the 3 men on the diagram below.

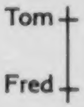


Original Problem

Tom is heavier than Fred but lighter than Marty. Write the names of the 3 men on the diagram below.

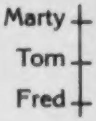
Problem Solution

Step 1. Tom is heavier than Fred . . . He would be placed above Fred on the diagram.



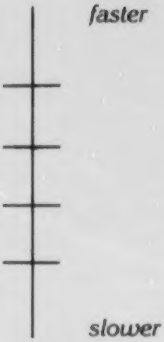
Step 2. . . . but lighter than Marty.

This says Tom is shorter than Marty. So Marty is placed above Tom on the diagram.



Problem 2

Jack is slower than Phil but faster than Val. Val is slower than Jack but faster than Pete. Write the names of the 4 men in order on the diagram below.

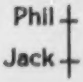


Original Problem

Jack is slower than Phil but faster than Val. Val is slower than Jack but faster than Pete. Write the names of the 4 men in order on the diagram below.

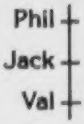
Problem Solution

Step 1. Jack is slower than Phil . . . He would be placed below Phil.



Step 2. . . . but faster than Val.

This says Jack is faster than Val. So Val is added below Jack.

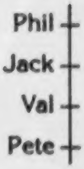


Step 3. Val is slower than Jack . . .

This is already represented in the diagram.

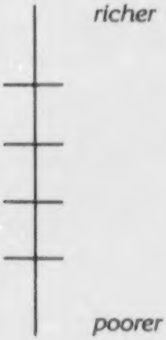
Step 4. . . . but faster than Pete.

Val is faster than Pete, so Pete is added to the diagram below Val.



Problem 3

If Bob and Fred are both richer than Tom, while Hal is poorer than Bob but richer than Fred, which man is poorest and which one is next poorest? Write the names of all 4 men in order on the diagram below.

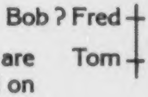


Original Problem

If Bob and Fred are both richer than Tom, while Hal is poorer than Bob but richer than Fred, which man is the poorest and which one is next poorest? Write the names of all 4 men in order on the diagram below.

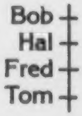
Problem Solution

Step 1. If Bob and Fred are both richer than Tom . . .



The problem does not indicate whether Bob and Fred are actually equal to each other. So they are represented on the diagram with a question mark between them, and with both of them above Tom.

Step 2. . . . while Hal is poorer than Bob but richer than Fred . . .



This means that Bob and Fred are not equal; Hal is between them, with Bob the tallest.

Problem 4

Paul and Tom are the same age. Paul is older than Cynthia. Cynthia is younger than Hal. Is Paul older or younger than Hal—or can this not be determined from the information?

Original Problem

Paul and Tom are the same age. Paul is older than Cynthia. Cynthia is younger than Hal. Is Paul older or younger than Hal—or can this not be determined from the information?

Problem Solution

Step 1. Paul and Tom are the same age. Paul is older than Cynthia.

This can be represented in a diagram.

Paul = Tom
Cynthia

Step 2. Cynthia is younger than Hal.

This says Hal is older than Cynthia. But it doesn't indicate whether Hal is older or younger than Paul and Tom. This is indicated with a bracket on the diagram.

$$\text{Paul} = \text{Tom}\overset{?}{\underset{?}{}}\Big\}\,\text{Hal}$$
Cynthia

Step 3. Is Paul older or younger than Hal—or can this not be determined from the information?

It cannot be determined.

Problem 5

Cathy knows French and German, Sandra knows Swedish and Russian, Cindy knows Spanish and French, Paula knows German and Swedish. If French is easier than German, Russian is harder than Swedish, German is easier than Swedish, and Spanish is easier than French, which girl knows the most difficult languages?

Hint: Begin by making a diagram showing the order of difficulty of the languages.

Original Problem

Cathy knows French and German, Sandra knows Swedish and Russian, Cindy knows Spanish and French, Paula knows German and Swedish. If French is easier than German, Russian is harder than Swedish, German is easier than Swedish, and Spanish is easier than French, which girl knows the most difficult languages?

Problem Solution

Step 1. Strategy For Beginning The Problem: The question asks which girl knows the most difficult languages. Therefore the first step is to order the languages by difficulty. This information is contained in the second sentence of the problem, so the solution starts with the second sentence.

Step 2. If French is easier than German . . .

This can be shown in a diagram. The easier language has been arbitrarily put below the harder language.

German
French

Step 3. . . . Russian is harder than Swedish.

This can be shown in a separate diagram.

Russian
Swedish

Step 4. . . . German is easier than Swedish . . .

This information shows how the two diagrams can be combined. German is placed below Swedish since it is easier.

Russian
Swedish
German
French

Step 5. . . . Spanish is easier than French.

This can be added to the diagram.

Russian
Swedish
German
French
Spanish

Step 6. The diagram shows that Russian and Swedish are the two most difficult languages. In order to answer the question it is necessary to find the girl who speaks these two languages. Scanning the first sentence shows that Sandra speaks them.

Sandra speaks Russian and Swedish, the most difficult languages.

Problem 6

Paul, Sam and Tom differ in height. Their last names are Smith, Jones and Calvin, but not necessarily in that order. Paul is taller than Tom but shorter than Sam. Smith is the tallest of the 3 and Calvin is the shortest. What are Paul's and Tom's last names?

Original Problem

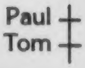
Paul, Sam and Tom differ in height. Their last names are Smith, Jones and Calvin, but not necessarily in that order. Paul is taller than Tom but shorter than Sam. Smith is the tallest of the 3 and Calvin is the shortest. What are Paul's and Tom's last names?

Problem Solution

Step 1. Strategy For Beginning the Problem: Information is given about the heights of the men in terms of their first names and in terms of their last names. By diagramming both sets of relationships, the first names can be matched up with the last names.

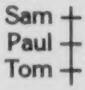
Step 2. Paul is taller than Tom . . .

This is represented in the diagram on the right.



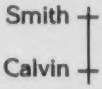
Step 3. . . . but shorter than Sam.

This says Paul is shorter than Sam, so Sam is added above Paul in the diagram.

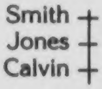


Step 4. Smith is the tallest of the 3 and Calvin is the shortest.

This information can be represented in a new diagram referring to the men's last names. A blank space is left for the third man's name.



Step 5. Scanning the problem shows that the third man's last name is Jones, so this can be added to the diagram.



Step 6. Comparing the 2 diagrams shows that:

Paul's last name is Jones;
Tom's last name is Calvin.

Problem 7

Three fathers—Pete, John and Nick—have between them a total of 15 children of which 9 are boys. Pete has 3 girls and John has the same number of boys. John has 1 more child than Pete, who has 4 children. Nick has 4 more boys than girls and the same number of girls as Pete has boys. How many boys each do Nick and Pete have?

Hint: It may be helpful to arrange the information into a table of the type shown below.

	Boys	Girls	Total
Pete			
John			
Nick			
Total			

Original Problem

Three fathers—Pete, John, and Nick—have between them a total of 15 children of which 9 are boys. Pete has 3 girls and John has the same number of boys. John has 1 more child than Pete who has 4 children. Nick has 4 more boys than girls and the same number of girls as Pete has boys. How many boys each do Nick and Pete have?

Problem Solution

Step 1. Three fathers—Pete, John and Nick—have between them a total of 15 children of which 9 are boys.

	Boys	Girls	Total
Pete			
John			
Nick			
Total	9		15

Step 2. The table shows that there are 9 boys and a total of 15 children. So there must be 6 girls.

	Boys	Girls	Total
Pete			
John			
Nick			
Total	9	6	15

Step 3. Pete has 3 girls and John has the same number of boys.

	Boys	Girls	Total
Pete		3	
John	3		
Nick			
Total	9	6	15

Step 4. John has 1 more child than Pete who has 4 children. This means John has 5 children.

	Boys	Girls	Total
Pete		3	4
John	3		5
Nick			
Total	9	6	15

Step 5. The table shows that Pete has 4 children of which 3 are girls, so he must have 1 boy. Also, John must have 2 girls; and Nick must have 6 children since Pete and John together have 9 children.

	Boys	Girls	Total
Pete	1	3	4
John	3	2	5
Nick			6
Total	9	6	15

Step 6. Looking at the last table shows that Pete and John together have 4 boys, so Nick must have 5 boys. Also, Nick must have 1 girl.

	Boys	Girls	Total
Pete	1	3	4
John	3	2	5
Nick	5	1	6
Total	9	6	15

Step 7. The table is filled in, but there is 1 sentence of information remaining. It can be read and checked against the table to make sure that the table is correct.

Nick has 4 more boys and the same number of girls as Pete has boys.

The table shows that Nick has 5 boys and 1 girl—so he does have 4 more boys than girls. The table is correct on this point. Also, Nick has 1 girl and Pete has 1 boy, so this is correct.

Step 8. How many boys each do Nick and Pete have?

The table shows: Nick has 5 boys.
Pete has 1 boy.

Problem 8

Paula, Joanne and Mary own a total of 16 dogs, among which are 3 poodles, twice that many cocker spaniels, and the remainder German shepherds and collies. Joanne despises poodles and collies, but owns 4 cocker spaniels and 2 German shepherds, giving her a total of 6 dogs. Paula owns 1 poodle and only 2 other dogs, both German shepherds. Mary owns 3 collies and several other dogs. What other dogs (and how many of each) does Mary own?

Note: In constructing a table for this problem, remember to enter zeros as well as positive numbers whenever appropriate. Some students forget to enter zeros and therefore think this problem cannot be solved. Also, enter totals as soon as you can. For example, enter the total of 16 dogs now.

Original Problem

Paula, Joanne and Mary own a total of 16 dogs, among which are 3 poodles, twice that many cocker spaniels, and the remainder German shepherds and collies. Joanne despises poodles and collies, but owns 4 cocker spaniels and 2 German shepherds, giving her a total of 6 dogs. Paula owns 1 poodle and only 2 other dogs, both German shepherds. Mary owns 3 collies and several other dogs. What other dogs (and how many of each) does Mary own?

Problem Solution

Step 1. Paula, Joanne and Mary own a total of 16 dogs, among which are 3 poodles, twice that many cocker spaniels and the remainder German shepherds and collies.

	Poodle	Spaniel	Shepherd	Collie	Total
Paula					
Joanne					
Mary					
Total	3	6			16

Step 2. Joanne despises poodles and collies, but owns 4 cocker spaniels and 2 German shepherds.

	Poodle	Spaniel	Shepherd	Collie	Total
Paula					
Joanne	0	4	2	0	6
Mary					
Total	3	6			16

Step 3. Paula owns 1 poodle and only 2 other dogs, both German shepherds.

	Poodle	Spaniel	Shepherd	Collie	Total
Paula	1	0	2	0	3
Joanne	0	4	2	0	6
Mary					
Total	3	6			16

Step 4. Since Paula and Joanne own 9 dogs between them, and there are a total of 16 dogs, Mary must own 7 dogs. Also, since Paula and Joanne have only 1 poodle between them, Mary must own 2 poodles. Again, Mary must own 2 spaniels since Paula and Joanne have 4 between them. This is shown in the next table.

	Poodle	Spaniel	Shepherd	Collie	Total
Paula	1	0	2	0	3
Joanne	0	4	2	0	6
Mary	2	2			7
Total	3	6			16

Step 5. Mary owns 3 collies and several other dogs.

	Poodle	Spaniel	Shepherd	Collie	Total
Paula	1	0	2	0	3
Joanne	0	4	2	0	6
Mary	2	2		3	7
Total	3	6			16

Step 6. The table shows there are a total of 3 collies. Thus there must be 4 shepherds. This means that Mary owns no shepherds.

	Poodle	Spaniel	Shepherd	Collie	Total
Paula	1	0	2	0	3
Joanne	0	4	2	0	6
Mary	2	2	0	3	7
Total	3	6	4	3	16

Step 7. What other dogs (and how many of each) does Mary own?

The table shows Mary owns 2 poodles and 2 cocker spaniels in addition to her collies.

Problem 9

Salesmen who work for the Acme Wig Company are assigned to a different city each year. Henry began working for Acme in New York in 1965, and in the succeeding 4 years worked in Minneapolis, New Haven, Youngstown and Charleston, in that order. Peter worked for Acme in New Haven in 1963, and in succeeding years worked in New York, Charleston, Minneapolis and Youngstown. Fred worked for Acme in Charleston in 1967; the previous 2 years he had worked first in New Haven and then in Minneapolis. John worked in Charleston in 1968. Before that he was in New Haven, before that Youngstown, and before that New York. Which Acme salesmen were in New Haven in 1967? Which ones were in Minneapolis in 1966?

Original Problem

Salesmen who work for the Acme Wig Company are assigned to a different city each year. Henry began working for Acme in New York in 1965, and in the succeeding 4 years worked in Minneapolis, New Haven, Youngstown and Charleston, in that order. Peter worked for Acme in New Haven in 1963, and in succeeding years worked in New York, Charleston, Minneapolis and Youngstown. Fred worked for Acme in Charleston in 1967; the previous 2 years he had worked first in New Haven and then in Minneapolis. John worked in Charleston in 1968. Before that he was in New Haven, before that Youngstown, and before that New York. Which Acme salesmen were in New Haven in 1967? Which ones were in Minneapolis in 1966?

Problem Solution

Two ways of organizing the information are shown in tables below:

	Henry	Peter	Fred	John
1963		New Haven		
1964		New York		
1965	New York	Charleston	New Haven	New York
1966	Minneapolis	Minneapolis	Minneapolis	Youngstown
1967	New Haven	Youngstown	Charleston	New Haven
1968	Youngstown			Charleston
1969	Charleston			

	Henry	Peter	Fred	John
New York	1965	1964		1965
Minneapolis	1966	1966	1966	
New Haven	1967	1963	1965	1967
Youngstown	1968	1967		1966
Charleston	1969	1965	1967	1968

Both tables show that Henry and John were in New Haven in 1967, and that Henry, Peter and Fred were in Minneapolis in 1966.

Problem 10

On a certain day I ate lunch at Tommy's, took out 2 books from the library (*The Sea Wolf* and *Martin Eden*, both by Jack London), visited the museum, and had a cavity filled. Tommy's is closed on Wednesday, the library is closed on weekends, the museum is only open Monday, Wednesday and Friday, and my dentist has office hours Tuesday, Friday and Saturday. On which day of the week did I do all these things?

Original Problem

On a certain day I ate lunch at Tommy's, took out 2 books from the library (*The Sea Wolf* and *Martin Eden*, both by Jack London), visited the museum, and had a cavity filled. Tommy's is closed on Wednesday, the library is closed on weekends, the museum is only open Monday, Wednesday and Friday, and my dentist has office hours Tuesday, Friday and Saturday. On which day of the week did I do all these things?

Problem Solution

Step 1. Suggestion for beginning the problem: The restrictions on when these activities occurred are stated in the second sentence.

Step 2. Tommy's is closed on Wednesday . . .

S M T ~~W~~ TH F SAT

Step 3. . . . the library is closed on weekends . . .

~~S~~ M T ~~W~~ TH F ~~SAT~~

Step 4. . . . the museum is only open Monday, Wednesday and Friday . . .

This means it is closed the other days.

~~S~~ M ~~T~~ ~~W~~ ~~TH~~ F ~~SAT~~

Step 5. . . . and my dentist has office hours Tuesday, Friday and Saturday.

This eliminates Monday.

~~S~~ ~~M~~ ~~T~~ ~~W~~ ~~TH~~ F ~~SAT~~

Step 6. On which day of the week did I do all these things?

Friday.

Problem 11

Boris, Irwin and Steven are engaged in the occupations of librarian, teacher and electrician, although not necessarily in that order. The librarian is Steven's cousin. Irwin lives next door to the electrician. Boris, who knows more facts than the teacher, must drive 45 minutes to visit Irwin's house.

What is each man's occupation?

It is helpful to use a table like the one shown below. Here is step 1 in the solution.

Step 1. The problem says the librarian is Steven's cousin. That means Steven is not the librarian. This is shown by writing "NO" in the table.

	librarian	teacher	electrician
Boris			
Irwin			
Steven	NO		

Complete the table and determine each man's occupation.

Original Problem

Boris, Irwin and Steven are engaged in the occupations of librarian, teacher and electrician, although not necessarily in that order. The librarian is Steven's cousin. Irwin lives next door to the electrician. Boris, who knows more facts than the teacher, must drive 45 minutes to visit Irwin's house.

Problem Solution

Step 1. The problem says the librarian is Steven's cousin. That means Steven is not the librarian. This is shown by writing "NO" in the table.

	librarian	teacher	electrician
Boris			
Irwin			
Steven	NO		

Step 2. Irwin lives next door to the electrician. This means Irwin is not the electrician.

	librarian	teacher	electrician
Boris			
Irwin			NO
Steven	NO		

Step 3. "Boris, who knows more facts than the teacher . . ." This means Boris is not the teacher.

	librarian	teacher	electrician
Boris		NO	
Irwin			NO
Steven	NO		

Step 4. ". . . must drive 45 minutes to visit Irwin's house."

An earlier sentence said Irwin lives next door to the electrician. Since Boris must drive 45 minutes to visit Irwin, he is not the electrician.

	librarian	teacher	electrician
Boris		NO	NO
Irwin			NO
Steven	NO		

Step 5. From the table we see that Boris must be the librarian.

	librarian	teacher	electrician
Boris	YES	NO	NO
Irwin			NO
Steven	NO		

Step 6. The table also shows that Steven must be the electrician.

	librarian	teacher	electrician
Boris	YES	NO	NO
Irwin			NO
Steven	NO		YES

Step 7. Irwin must be the teacher.

Problem 12

Three men—Fred, Ed and Ted—are married to Joan, Sally and Vickie, but not necessarily in that order. Joan, who is Ed's sister, lives in Detroit. Fred dislikes animals. Ed weighs more than the man who is married to Vickie. The man married to Sally breeds Siamese cats as a hobby. Fred commutes over 200 hours a year from his home in Ann Arbor to his job in Detroit. Match up the men with the women they married.

Original Problem

Three men—Fred, Ed and Ted—are married to Joan, Sally and Vickie, but not necessarily in that order. Joan, who is Ed's sister, lives in Detroit. Fred dislikes animals. Ed weighs more than the man who is married to Vickie. The man married to Sally breeds Siamese cats as a hobby. Fred commutes over 200 hours a year from his home in Ann Arbor to his job in Detroit. Match up the men with the women they married.

Problem Solution

Step 1. The problem says Joan is Ed's sister. Therefore Joan and Ed are not married.

	Joan	Sally	Vickie
Fred			
Ed	NO		
Ted			

Step 2. The problem says Joan lives in Detroit, and Fred dislikes animals. There is no way to use that information yet.

The next statement says Ed weighs more than the man who is married to Vickie. That means Ed is not married to Vickie.

	Joan	Sally	Vickie
Fred			
Ed	NO		NO
Ted			

Step 3. The table shows that Ed must be married to Sally.

	Joan	Sally	Vickie
Fred			
Ed	NO	YES	NO
Ted			

Step 4. Since Ed is married to Sally, we know that neither Fred nor Ted is married to Sally.

	Joan	Sally	Vickie
Fred		NO	
Ed	NO	YES	NO
Ted		NO	

Step 5. The problem says that the man married to Sally breeds Siamese cats as a hobby. Earlier the problem said that Fred dislikes animals. That means that Fred is not married to Sally. This is already shown in the table.

Step 6. The problem says that Fred commutes from his home in Ann Arbor to his job in Detroit. Earlier it said Joan lives in Detroit. Therefore we can conclude that Fred is not married to Joan.

	Joan	Sally	Vickie
Fred	NO	NO	
Ed	NO	YES	NO
Ted		NO	

Step 7. Fred must be married to Vickie.

Joan must be married to Ted.

Problem 13

You are facing east, you make an about-face, and then you turn left. Which direction is now on your left side?

Original Problem

You are facing east, you make an about-face, and then you turn left. Which direction is now on your left side?

Problem Solution

Step 1. A useful aid in solving a problem like this in your head (without making a diagram) is to picture yourself standing on some map with which you are familiar. The following solution uses a map of the United States.

Step 2. You are facing east . . .

On a map of the United States you would be facing the Atlantic Ocean, with Canada on your left and the southern states such as Florida and Texas on your right.

Step 3. . . . you make an about-face . . .

You might turn left, past Canada, and face California.

Step 4. . . . and then you turn left.

You turn south facing Texas.

Step 5. Which direction is now on your left side?

If you are facing south, the east coast is on your left.

East is on your left.

Problem 14

A train left city *A* at 9:35 and arrived at city *B* 5 hours and 40 minutes later. What time did it arrive at city *B*?

Original Problem

A train left city *A* at 9:35 and arrived at city *B* 5 hours and 40 minutes later. What time did it arrive at city *B*?

Problem Solution

Step 1. 9:35 plus 3 hours is 12:35.

Step 2. 12:35 plus 2 more hours is 2:35.

Step 3. 2:35 plus 40 minutes is 2:75.

Step 4. 2:75 is 3:15. (Since there are 60 minutes in 1 hour.) The train arrived at 3:15.

Problem 15

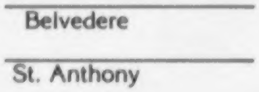
Belvedere Street is parallel to St. Anthony Street. Davidson Street is perpendicular to River Street. River Street is parallel to St. Anthony Street. Is Davidson Street parallel or perpendicular to Belvedere?

Original Problem

Belvedere Street is parallel to St. Anthony Street. Davidson Street is perpendicular to River Street. River Street is parallel to St. Anthony Street. Is Davidson Street parallel or perpendicular to Belvedere?

Problem Solution

Step 1. Belvedere Street is parallel to St. Anthony Street.

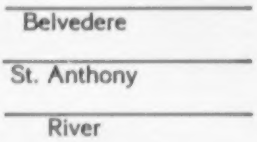


Step 2. Davidson Street is perpendicular to River Street.

This sentence presents information on 2 new streets. It can be skipped over temporarily.

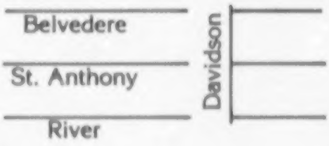
Step 3. River Street is parallel to St. Anthony Street.

This can be added to the above diagram.



Step 4. Now, going back to the second sentence: Davidson Street is perpendicular to River Street.

This can now be added to the diagram.



Step 5. Is Davidson Street parallel or perpendicular to Belvedere?

The diagram shows that Davidson is perpendicular.

Problem 16

In the town of Pottsville, streets with names that begin with a vowel and end with a consonant run north-south. Those which begin with a consonant and end with a vowel run east-west. Others may run either way. If Carter Street is perpendicular to Agnes Street, is it parallel or perpendicular to Sheridan Street which runs north-south?

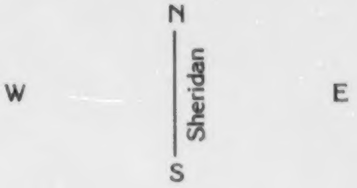
Original Problem

In the town of Pottsville, streets with names that begin with a vowel and end with a consonant run north-south. Those which begin with a consonant and end with a vowel run east-west. Others may run either way. If Carter Street is perpendicular to Agnes Street, is it parallel or perpendicular to Sheridan Street which runs north-south?

Problem Solution

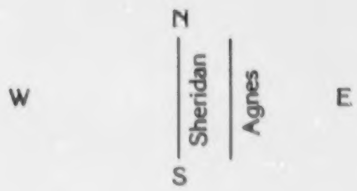
Step 1. The question concerns the relationship between Carter and Sheridan.

Step 2. Sheridan runs north-south.

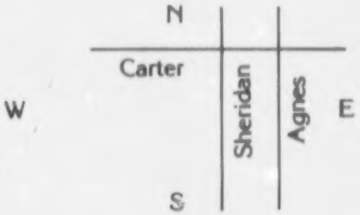


Step 3. Carter begins with a consonant and ends with a consonant so it can run either way.

Step 4. Agnes begins with a vowel and ends with a consonant so it runs north-south.



Step 5. The problem says Carter is perpendicular to Agnes.

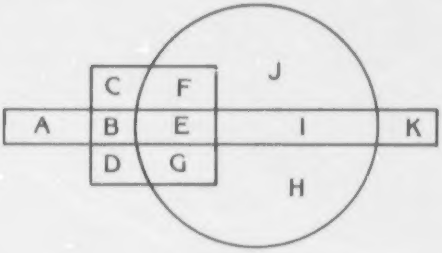

N
Carter
W
Sheridan
Agnes
E
S

Step 6. The diagram shows that Carter Street is perpendicular to Sheridan.

Problem 17

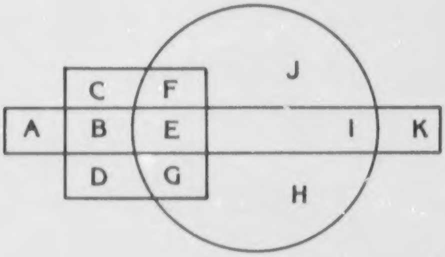
How many letters are in either the rectangle or the square, but not in both?

Note: The problem says nothing about the circle. Therefore treat the circle as if were not there. A simple example illustrates this idea. If an instructor asked all blue-eyed students to stand up, it would mean both *tall* and *short* blue-eyed students. Since height is not mentioned, it should be ignored in this situation.



Original Problem

How many letters are in either the rectangle or the square, but not in both?



Problem Solution

A is in just the rectangle.

C is in just the square.

D is in just the square.

G is in just the square (ignoring the circle).

F is in just the square (ignoring the circle).

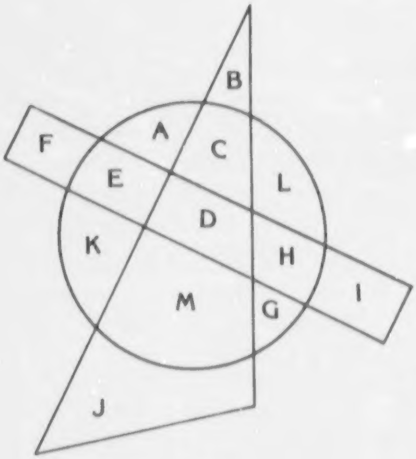
I is in just the rectangle (ignoring the circle).

K is in just the rectangle.

There are 7 letters in either the rectangle or the square, but not in both.

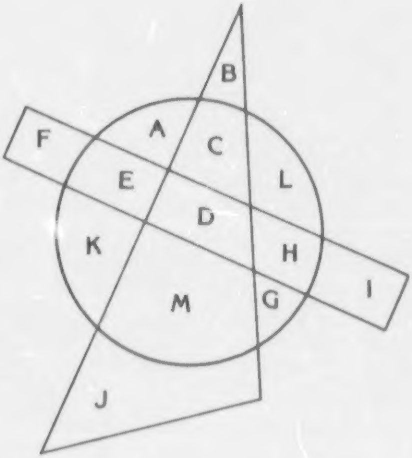
Problem 18

In working this problem, consider the diagram to be made up of 3 major geometrical figures. *1.* a triangle; *2.* a circle; and *3.* a rectangle. How many letters are in exactly 2 (but not 3) of these figures?



Original Problem

In working this problem, consider the diagram to be made up of 3 major geometrical figures: *1*. a triangle; *2*. a circle; and *3*. a rectangle. How many letters are in exactly 2 (but not 3) of these figures?



Problem Solution

E is in the circle and the rectangle.

C is in the triangle and the circle.

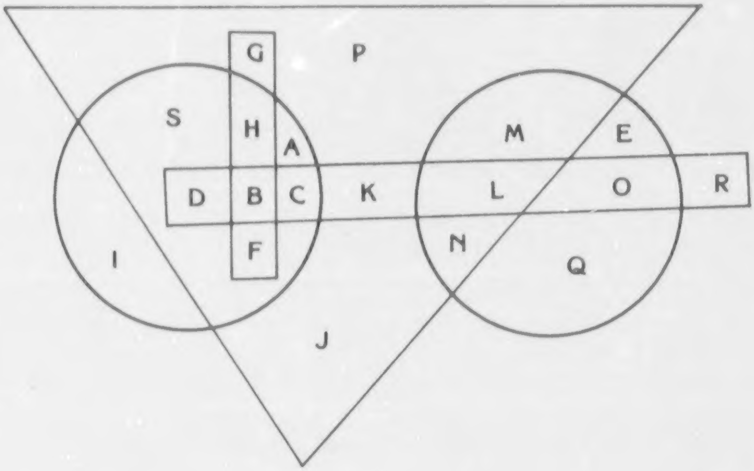
H is in the circle and the rectangle.

M is in the circle and the triangle.

There are 4 letters in exactly 2 figures.

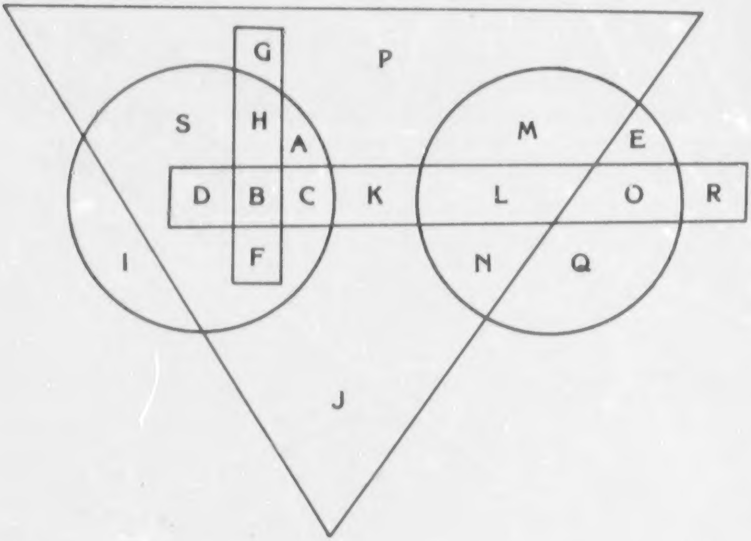
Problem 19

In working this problem, consider the diagram to be made up of 5 major geometrical figures: *1.* 1 triangle; *2.* 2 circles; and *3.* 2 rectangles. List the letters which are in the same number of major geometrical figures as the letter *G*. List them here ____________ .



Original Problem

In working this problem, consider the diagram to be made up of 5 major geometrical figures: *1.* 1 triangle; *2.* 2 circles; and *3.* 2 rectangles. List the letters which are in the same number of major geometrical figures as the letter *G*. List them here ____________ .



Problem Solution

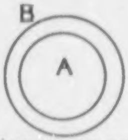
Step 1. The letter *G* is in the triangle and also in 1 of the rectangles. So it is in 2 major geometrical figures.

Step 2. *S* is in the triangle and 1 of the circles.
A is in the triangle and 1 of the circles.
K is in the triangle and 1 of the rectangles.
N is in the triangle and 1 of the circles.
M is in the triangle and 1 of the circles.
O is in 1 of the rectangles and 1 of the circles.

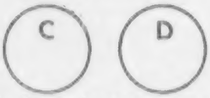
Problem 20

The next group of problems employ Venn diagrams. Here is the way Venn diagrams can be used to represent certain statements.

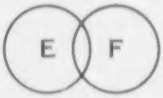
Example *a*. All A are B. (For instance, all dogs are animals.)



Example *b*. No C are D. (For instance, no people are cars.)



Example *c*. Some E are F. (For instance, some women are democrats.)



Problem: Make a Venn diagram with 3 circles showing the following relationships.

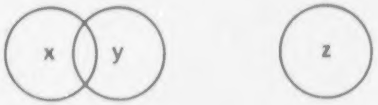
Some x are y. No x are z. No y are z.

Original Problem

Make a Venn diagram with 3 circles showing the following relationships.

Some x are y. No x are z. No y are z.

Problem Solution



Problem 21

Make a Venn diagram with 3 circles showing the following relationships.

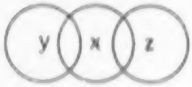
Some x are y. Some x are z. No y are z.

Original Problem

Make a Venn diagram with 3 circles showing the following relationships.

Some x are y. Some x are z. No y are z.

Problem Solution



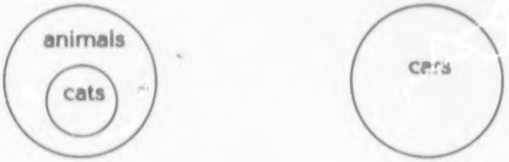
Problem 22

Make a Venn diagram showing the relationships between cats, animals and cars, using one circle to represent cats, another to represent animals, and a third to represent cars.

Original Problem

Make a Venn diagram showing the relationships between cats, animals and cars, using one circle to represent cats, another to represent animals, and a third to represent cars.

Problem Solution



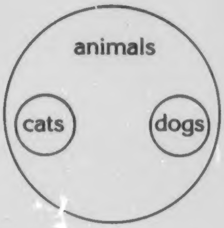
Problem 23

Make a Venn diagram showing the relationship between cats, dogs and animals.

Original Problem

Make a Venn diagram showing the relationships between cats, dogs and animals.

Problem Solution


animals
cats
dogs

Problem 24

If some of Totteville's residents have brown eyes and some of Totteville's residents are women, is the statement "Some of Totteville's residents are brown-eyed women" true, false, or unsubstantiated?

Original Problem

If some of Totteville's residents have brown eyes and some of Totteville's residents are women, is the statement "Some of Totteville's residents are brown-eyed women" true, false, or unsubstantiated?

Problem Solution

From the statement we can't tell whether none of the brown-eyed residents are women (as shown in diagram 1) or whether some of the brown-eyed residents are women (as shown in diagram 2).

Therefore the statement is not definitely false (diagram 1) or definitely true (diagram 2). It is just unsubstantiated.

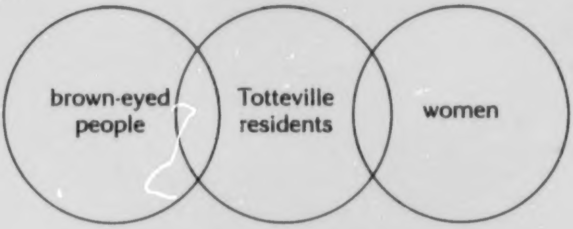

brown-eyed people
Totteville residents
women

Diagram 1

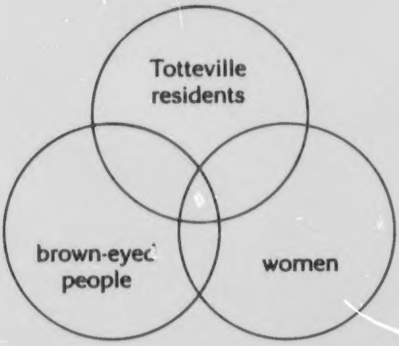

Totteville residents
brown-eyed people
women

Diagram 2

Problem 25

For this problem assume the first two statements are correct and make a diagram to represent the relationships. Then answer the questions.

All bears are butterflies. All bees are bears.

a. Can you be certain that all bees are butterflies?

b. Can you be certain that all butterflies are bees?

Original Problem

For this problem assume the first two statements are correct and make a diagram to represent the relationships. Then answer the questions.

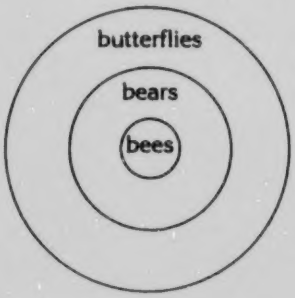
All bears are butterflies. All bees are bears.

a. Can you be certain that all bees are butterflies?

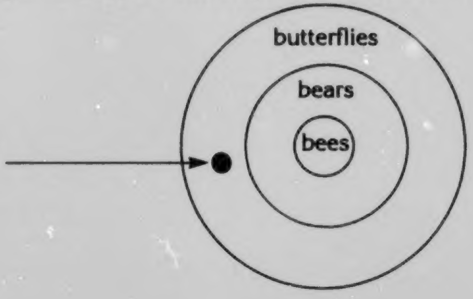
b. Can you be certain that all butterflies are bees?

Problem Solution

Here is the diagram showing that all bees are bears and all bears are butterflies.



Answer for question a.	The diagram shows that all bees are butterflies. Anything in the circle for bees is automatically in the circle for butterflies.
Answer for question b.	The diagram shows that there can be butterflies which are not bees. For example, the dot in the diagram below is a butterfly that is not a bee.



This is a butterfly that is not a bee.

Problem 26

The fire department wants to send booklets on fire hazards to all teachers and homeowners in town. How many booklets does it need, using these statistics? Use a Venn diagram in solving this problem.

Homeowners	50,000
Teachers	4,000
Teachers who own their homes	3,000

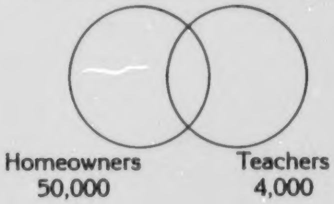
Original Problem

The fire department wants to send booklets on fire hazards to all teachers and homeowners in town. How many booklets does it need, using these statistics?

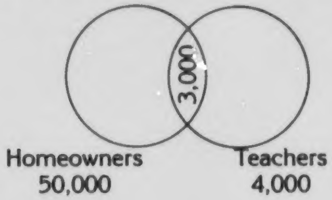
Homeowners	50,000
Teachers	4,000
Teachers who own their homes	3,000

Problem Solution

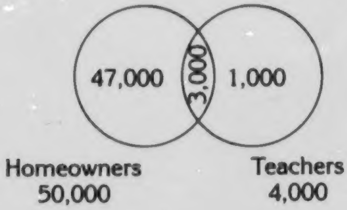
Step 1. The left circle is the homeowners. The right circle is the teachers.


Homeowners
50,000
Teachers
4,000

Step 2. Three thousand people are both homeowners and teachers.


3,000
Homeowners
50,000
Teachers
4,000

Step 3. The remaining portions of the circles can be filled in.


47,000
3,000
1,000
Homeowners
50,000
Teachers
4,000

Step 4. The total booklets needed is:

$$\begin{array}{r} 47{,}000 \\ 3{,}000 \\ \underline{1{,}000} \\ 51{,}000 \end{array}$$

Problem 27

An insurance company wants to contact all physicians and all licensed drivers in town. Using these statistics, how many people must be contacted? Draw a Venn diagram with your solution.

Licensed drivers	8,000
Physicians	750
Physicians with drivers licenses	750

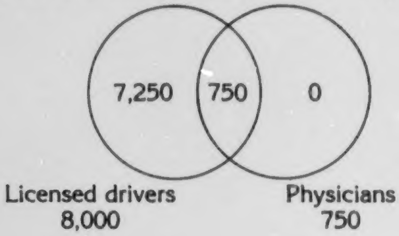
Original Problem

An insurance company wants to contact all physicians and all licensed drivers in town. Using these statistics, how many people must be contacted?

Licensed drivers	8,000
Physicians	750
Physicians with drivers licenses	750

Problem Solution

Step 1. Seven hundred and fifty licensed drivers are physicians.


7,250
750
0
Licensed drivers
8,000
Physicians
750

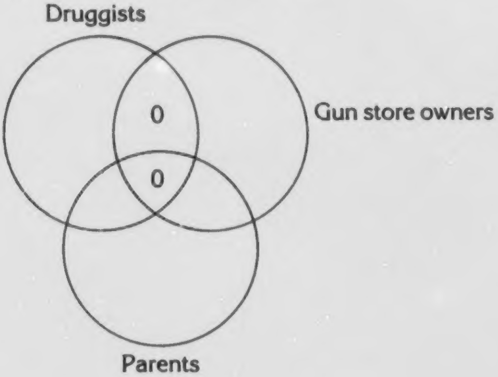
Step 2. 8,000 people need to be contacted.

Problem 28

The government wants to contact all druggists, all gun store owners, and all parents in a town. How many people must be contacted, using these statistics?

Druggists	10
Gun store owners	5
Parents	3,000
Druggists who own gun stores	0
Druggists who are parents	7
Gun store owners who are parents	3

Hint: Use this Venn diagram. Note that since no druggists own gun stores, two sections have zeroes.



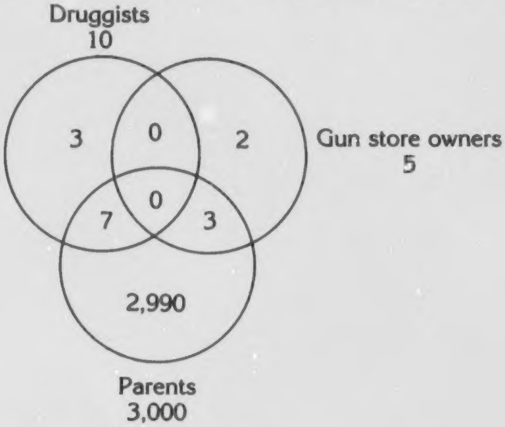
Original Problem

The government wants to contact all druggists, all gun store owners, and all parents in a town. How many people must be contacted, using these statistics?

Druggists	10
Gun store owners	5
Parents	3,000
Druggists who own gun stores	0
Druggists who are parents	7
Gun store owners who are parents	3

Problem Solution

Step 1. The diagram shows the number of people in each category.


Druggists
10
3
0
2
Gun store owners
5
0
7
3
2,990
Parents
3,000

Step 2. The total number that must be contacted is:

$$\begin{array}{r} 2{,}990 \\ 7 \\ 3 \\ 2 \\ +\ 3 \\ \hline 3{,}005 \end{array}$$

Problem 29

If deleting the letters B, R and A from the word *burglary* leaves a meaningful 3-letter word, circle the first R in this word burglary. Otherwise circle the U in the word burglary where the word appears for the third time in the exercise.

Original Problem

If deleting the letters B, R and A from the word *burglary* leaves a meaningful 3-letter word, circle the first R in this word burglary. Otherwise circle the U in the word burglary where the word appears for the third time in the exercise.

Problem Solution

Step 1. If deleting the letters B, R and A from the word burglary . . .

burglary

Step 2. . . . leaves a meaningful 3-letter word . . .

No. The word which remains is *ugly,* and it has four letters.

Step 3. . . . circle the first R in this word *burglary.*

So this should not be done.

Step 4. Otherwise circle the U in the word *burglary* where the word appears for the third time in the exercise.

The word *burglary* occurs for the third time in [illegible]e last sentence of the problem. The U has been circled on the original problem above.

Problem 30

Indicate the position of the letter in the word *enrage* which is the seventh letter in the alphabet.

a. First
b. Second
c. Third
d. Fourth
e. Fifth
f. Sixth
g. Seventh
h. Eighth

Original Problem

Indicate the position of the letter in the word *enrage* which is the seventh letter in the alphabet.

a. First
b. Second
c. Third
d. Fourth
e. [✓ Fifth]
f. Sixth
g. Seventh
h. Eighth

Problem Solution

Step 1. The seventh letter in the alphabet is: A B C D E F G.

Step 2. The letter *G* is the fifth letter in the word *enrage:* enrage.
↑

Step 3. The word *fifth* is circled above.

Problem 31

What number is twice the distance below 20 as 7 is above 4? Circle your answer below.

27 26 25 23 22 18 17 15 14 13

Original Problem

What number is twice the distance below 20 as 7 is above 4? Circle your answer below.

27 26 25 23 22 18 17 15 [✓ 14] 13

Problem Solution

Step 1. The distance that 7 is from 4 is: 7 − 4 = 3.

Step 2. Twice this distance is: 2 *x* 3 = 6.

Step 3. The number that is 6 below 20 is: 20 − 6 = 14.

The number *14* is circled in the original problem above.

Problem 32

If the fourth number is greater than the second number, circle the third number unless the third number is greater than the fifth number. In this case, circle the number which is the difference between the second number and the seventh number.

8 4 6 5 2 1 9

Original Problem

If the fourth number is greater than the second number, circle the third number unless the third number is greater than the fifth number. In this case, circle the number which is the difference between the second number and the seventh number.

8 4 6 [✓ 5] 2 1 9

Problem Solution

Step 1. If the fourth number is greater than the second number . . .

The fourth number is *5*. The second number is *4*. So the fourth number is greater than the second number.

Step 2. . . . circle the third number unless the third number is greater than the fifth number.

The third number is *6*. The fifth number is *2*. So the third number is greater than the fifth number and, therefore, it should not be circled.

Step 3. In this case, circle the number which is the difference between the second number and the seventh number.

The second number is *4*. The seventh number is *9*. The difference between 9 and 4 is 5. Therefore the *5* has been circled on the original problem above.

Problem 33

If the difference between the second and the fourth numbers is greater than the difference between the third and the fifth numbers, circle the seventh number. Otherwise, obtain the difference between the differences and circle it.

3 3 4 6 9 7 9 2

Original Problem

If the difference between the second and the fourth numbers is greater than the difference between the third and the fifth numbers, circle the seventh number. Otherwise, obtain the difference between the differences and circle it.

3 3 4 6 9 7 9 ②

Problem Solution

Step 1. If the difference between the second and the fourth number . . .

The second number is *3*. The fourth number is *6*. The difference is 3.

Step 2. . . . is greater than the difference between the third and fifth numbers . . .

The third number is *4*. The fifth number is *9*. The difference is 5.

Therefore, the difference between the second and fourth numbers is not greater than the difference between the third and the fifth number.

Step 3. . . . circle the seventh number.

This should not be done.

Step 4. Otherwise, obtain the *difference between the differences* and circle it.

The 2 differences are 3 and 5. The difference between these two numbers is 2. Therefore, the difference between the differences is 2.

The *2* has been circled on the original problem above.

Problem 34

Circle the letter in the name *Anthony* which is 3 letters before the letter that follows the middle letter of the name.

Original Problem

Circle the letter in the name *Aⓝthony* which is 3 letters before the letter that follows the middle letter of the name.

Problem Solution

Step 1. The middle letter in Anthony is *H: Anthony.*

Step 2. The letter which follows *H* is *O: Anthony.*

Step 3. The letter which is 3 letters before *O* is *N: Anthony.*

Therefore *N* has been circled on the original problem above.

Problem 35

Cross out the letter in the word *participate* which is 2 letters before the second *T*.

Original Problem

Cross out the letter in the word *partici~~p~~ate* which is 2 letters before the second *T*.

Problem Solution

Step 1. The second *T* in *participate* is the one next to the *E*: *participate*.

Step 2. Two letters before this *T* is *P*: *participate*.

This *P* has been crossed out on the original problem above.

Problem 36 A Problem In Code Breaking

In a foreign language *lev klula buj* means "buy green peppers." However, words in this tongue are not always spoken in the same order as in English. For example, *buj* does not mean "peppers." Also, *ajm buj gyst* means "big green cars" and *lkuka lev ajm* means "quickly buy cars." How would you say *big peppers* in this tongue?

Hint: Write all 3 foreign phrases with their English translations next to them, then compare the phrases to see the words they have in common.

a. *buj klula* b. *klula buj* c. *klula gyst*
d. *lev gyst* e. *lkuka ajm*

Original Problem

In a foreign language *lev klula buj* means "buy green peppers." However, words in this tongue are not always spoken in the same order as in English. For example, *buj* does not mean "peppers." Also, *ajm buj gyst* means "big green cars" and *lkuka lev ajm* means "quickly buy cars." How would you say *big peppers* in this tongue?

Hint: Write all 3 foreign phrases with their English translations next to them, then compare the phrases to see the words they have in common.

a. *buj klula* b. *klula buj* c. *klula gyst*
d. *lev gyst* e. *lkuka ajm*

Problem Solution

Step 1. Three phrases are given in both the foreign tongue and English. These are:

(1) *lev klula buj*—"buy green peppers"
(2) *ajm buj gyst*—"big green cars"
(3) *lkuka lev ajm*—"quickly buy cars"

Step 2. The foreign word (1) and (2) have in common is *buj;* the English word they have in common is *green.* So *buj* must mean *green.*

Step 3. Apparently the foreign tongue has a different grammar than English since *green* is the second word in both (1) and (2), but *buj* is not the second word in (1) and (2).

Step 4. The foreign word that (1) and (3) have in common is *lev;* the English word they have in common is *buy.* So *lev* must mean *buy.*

Step 5. The foreign word that (2) and (3) have in common is *ajm;* the English word they have in common is *cars.* So *ajm* must mean *cars.*

Step 6. The problem asks for the translation of "big peppers." The 3 words which have already been translated are:

green—buj	*buy—lev*	*cars—ajm*

Step 7. The word *peppers* is contained in (1). Since *lev* and *buj* are already known to be *buy* and *green, klula* must be *peppers.*

Step 8. The word *big* is contained in (2). Since *buj* and *ajm* are already known to be *green* and *cars, gyst* must be *big.*

klula—peppers	*gyst—big*

Step 9. Therefore, "big peppers" must have the words *klula* and *gyst.* This is answer c.

Problem 37

In a different language *luk eir lail* means "heavy little package," *bo lail* means "heavy man" and *luk jo* means "pretty package." How would you say "little man" in this language?

Original Problem

In a different language *luk eir lail* means "heavy little package," *bo lail* means "heavy man" and *luk jo* means "pretty package." How would you say "little man" in this language?

Problem Solution

Step 1. *Luk eir lail* means "heavy little package;" *bo lail* means "heavy man."

The foreign word the 2 phrases have in common is *lail;* the English word they have in common is *heavy.* So apparently *lail* means *heavy.*

Step 2. The phrase *bo lail* means "heavy man," and since *lail* means *heavy, bo* must mean *man.*

Step 3. *Luk eir lail* means "heavy little package;" *luk jo* means "pretty package."

The foreign word they have in common is *luk;* the English word they have in common is *package.* So *luk* must mean *package.*

Step 4. *Luk eir lail* means "heavy little package." Since *luk* means *package,* and *lail* means *heavy, eir* must mean *little.*

Step 5. How would you say "little man" in this language?

It has been determined that: *Bo* means *man; Eir* means *little.*

Step 6. The foreign language apparently expresses certain ideas in the reverse order of English—adjectives follow nouns.

Therefore, "little man" is: *bo eir.*

Problem 38

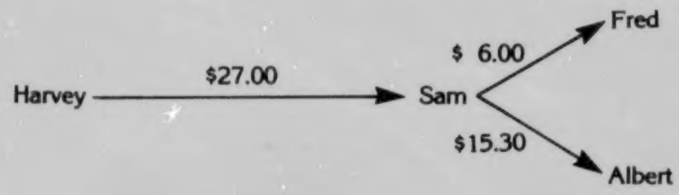
Harvey owes Sam $27.00. Sam owes Fred $6.00 and Albert $15.30. If, with Sam's permission, Harvey pays off Sam's debt to Albert, how much does he still owe Sam?

Original Problem

Harvey owes Sam \$27.00. Sam owes Fred \$6.00 and Albert \$15.30. If, with Sam's permission, Harvey pays off Sam's debt to Albert, how much does he still owe Sam?

Problem Solution

Step 1. The diagram below shows who owes whom money. The arrows point in the direction in which the money is owed.



Step 2. Harvey pays off Sam's debt to Albert, so this is subtracted from the amount he owes Sam.

$$\begin{array}{r} \$27.00 \\ -\ 15.30 \\ \hline \$11.70 \end{array}$$

Step 3. Harvey still owes Sam \$11.70.

Problem 39

Sally loaned $7.00 to Betty. But Sally borrowed $15.00 from Estella and $32.00 from Joan. Moreover, Joan owes $3.00 to Estella and $7.00 to Betty. One day the girls got together at Betty's house to straighten out their accounts. Which girl left with $18.00 more than she came with?

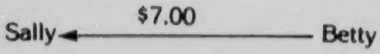
Hint: On your diagram, use arrows to show which person has to return money to which other person. Show the direction in which the money must be returned.

Original Problem

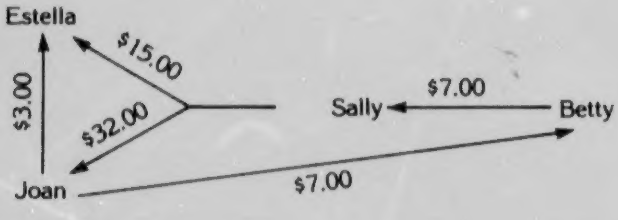
Sally loaned $7.00 to Betty. But Sally borrowed $15.00 from Estella and $32.00 from Joan. Moreover, Joan owes $3.00 to Estella and $7.00 to Betty. One day the girls got together at Betty's house to straighten out their accounts. Which girl left with $18.00 more than she came with?

Problem Solution

Step 1. The problem began "Sally loaned $7.00 to Betty." This is shown below with the arrow indicating the direction to which the money must be returned.



Step 2. The entire pattern of debts is shown in the following diagram.



Step 3. Which girl left with $18.00 more than she came with?

The diagram shows that Estella received $3.00 from Joan and $15.00 from Sally, for a total of $18.00.

Estella left with $18.00 more than she came with.

Problem 40

Lester has 12 times as many marbles as Kathy. John has half as many as Judy. Judy has half as many as Lester. Kathy has 6 marbles. How many marbles each do Lester and John have? You do not need to use algebra to solve this problem.

Original Problem

Lester has 12 times as many marbles as Kathy. John has half as many as Judy. Judy has half as many as Lester. Kathy has 6 marbles. How many marbles each do Lester and John have?

Problem Solution

Step 1. Kathy has 6 marbles.

Step 2. Lester has 12 times as many marbles as Kathy. So Lester has 72 (12 x 6) marbles.

In mathematical shorthand this can be written:

Lester's marbles = 12 x Kathy's marbles
Lester's marbles = 12 x 6
Lester's marbles = 72

Step 3. Judy has half as many as Lester. So Judy has ½ of 72 = 36.

This can be written:

Judy's marbles = ½ x Lester's marbles
Judy's marbles = ½ x 72
Judy's marbles = 36

Step 4. John has half as many as Judy. So John has ½ of 36 = 18.

Step 5. Lester has 72 marbles; John has 18.

ADDITIONAL PROBLEMS

1. Betty is shorter than Sally. Cynthia is taller than Sally. Carla is shorter than Betty. Is Sally shorter or taller than Carla?

2. John is faster than Pete. Dave is faster than Harvey. Dave is slower than Pete. Which man is fastest and which is slowest?

3. If Bob and Fred are both taller than Tom, while Hal is taller than Bob but shorter than Fred, which man is tallest and which is second tallest?

4. Gladys is a teacher, Sally a truck driver, Violet a crane operator, and Hannah a Hollywood stuntwoman. The truck driver is heavier than Hannah. The crane operator is lighter than the stuntwoman. Gladys is heavier than the truck driver. Which woman is heaviest and which is lightest?

5. Dracula hates daylight more than Wolfman unless Frankenstein hates daylight more than Dracula. In that case Wolfman hates daylight more than Dracula but less than Mummy. Mummy hates daylight more than Dracula but less than Frankenstein. Show a diagram of the monsters ordered according to their hatred of daylight.

6. A graph breaking down the cost of education for the state showed that the category labeled "operation, maintenance and auxiliary agencies" took a greater portion of the budget than "capital outlay." The category labeled "instruction" had the highest portion of the budget, while "interest" had a smaller portion than "capital outlay" and "general control" had a smaller portion than "interest." Show a diagram of the categories ordered according to their portion of the budget.

7. The Great Lakes differ in both their areas (measured in square miles) and their depths. However these two dimensions do not keep step perfectly. For example, Lake Michigan is exceeded in depth only by Lake Superior, but it is exceeded in area by both Lakes Superior and Huron. Lake Superior is by far the largest and deepest of the Great Lakes, but Lake Ontario, which is the smallest in area, is deeper than both Lakes Huron and Erie. Lake Erie is larger than Lake Ontario but it is not only shallower than Huron; it is also shallower than Ontario. Show the order of the Great Lakes according to depth.

8. Mr. and Mrs. Smith and Mr. and Mrs. Jones together have a total income of $800.00 a week, of which $180.00 is made by Mr. Smith. Mr. and Mrs. Smith together make $300.00 a week while Mr. Smith and Mr. Jones together make $450.00 a week. How much do Mrs. Jones and Mrs. Smith together make?

9. Three women—Pat, Joan, and Mary—have between them a total of 30 dresses of which 15 are cotton and the rest are either wool or synthetic fibre. Pat has 3 cotton dresses and 3 synthetics. Mary, who has a total of 8 dresses, has 4 cotton dresses. Pat has the same number of wool dresses as Mary has cotton dresses. Joan has as many wool dresses as Pat has cotton dresses. And Mary also has as many wool dresses as Pat has cotton dresses. How many total dresses does Joan have? (Show the table completely filled in.)

10. Four gangland hit men, Fred, John, Al and Nick, own between them 86 weapons made up of knives, handguns and rifles. Fred likes to be close to his work, so he owns 8 knives, 13 handguns, but no rifles. John is just the opposite. He prefers to remain at a distance from violence and therefore owns only high-powered rifles, 33 in all. Al is an all-around professional who owns 5 knives, 8 rifles, and various handguns. Nick, also a man of many talents, has a total of 15 weapons, among which are 3 knives and 5 handguns. Interestingly, among them the men own more than twice as many rifles as handguns, 48 rifles in all. How many handguns does Al own?

11. In this problem the "first quarter of the year" means January, February and March. The "second quarter" is April, May and June, and so on.

    Acme Realtors sold 23 houses during the first quarter of the year and again during the last quarter. Sales during the middle two quarters were not quite as good, so that the annual sales total was 57 houses. B & B Realty company sold 50 houses during the second quarter and half that many during the fourth quarter, for a total annual sales of 75 houses. Arco sold as many houses during the third quarter as B & B sold during the entire year, but during the other three quarters they did no better than B & B during the first quarter. Together the three companies sold 79 houses during the third quarter. How many total houses did the three companies sell during the second quarter?

12. John, Harry and Phil are married to Sally, Nancy and Arlene, but not necessarily in that order. John, who is Sally's brother, has five

children. Nancy, who is a certified public accountant, wants to wait several years before starting a family. Harry is married to John's sister. Who is Phil married to?

13. Judy, Celia and Betty are a math teacher, a truck driver and a housewife, but not necessarily in that order. Judy can't drive and is married to the brother of the math teacher. Celia is the best friend of the truck driver. Betty had a bad experience with math in grade school and has avoided all contact with math since then. What is each woman's occupation?

14. At Ajax Plastics the shipping clerk, the stockgirl, the saleswoman and the cashier are Rose, Hannah, Geraldine and Mary Jo, but Mr. Bigwig, the president of Ajax, can't remember who is which. Bigwig does, however, know these facts: Hannah likes both the saleswoman and the cashier; Mary Jo rides to work with the saleswoman and the cashier; the shipping clerk comes to work alone; Rose is slightly jealous of the cashier. Match up the women with their occupations.

15. You are facing south. You turn left, make an about-face, turn right, and turn right again. Which direction is behind you?

16. You are facing northeast and make an about-face. Which direction is on your right?

17. A man went to bed at 10:25 P.M. and arose at 4:10 A.M. How long was he in bed?

18. In Boontown streets that begin with a vowel run east-west unless they also end in a vowel in which case they run north-south. Other streets can go either way. Berkeley street is perpendicular to Alice street. In which direction does Berkeley run?

19. In a city known as the Big Carrot, streets that begin with a vowel and end with a consonant run east-west while those that begin with a consonant and end with a vowel run north-south. Other streets can go either way. A car driving north makes a left turn. Is it now traveling parallel or perpendicular to Eric street?

20. There is a car traveling north in the Big Carrot, the city described in problem 19. Is the car traveling parallel or perpendicular to Washington street which is perpendicular to Rose street?

21. Make a Venn diagram with 3 circles showing these relationships.

    All x are y. Some y are z. No x are z.

22. Make a Venn diagram with 3 circles showing these relationships.

    All x are y. Some z are x. Some z are not y. Some z are y but not x.

23. Make a Venn diagram with 4 circles showing these relationships.

    All x are y. No y are w. All v are w.

24. Make a Venn diagram showing the relationships among animals, birds, and ducks, using one circle to represent animals, another for birds, and a third for ducks.

25. Make a Venn diagram showing the relationships among animals, brown animals, and dogs, using one circle to represent animals, another for brown animals, and a third for dogs.

26. On Halloween the Big Pumpkin visits the homes of all children, all grandmothers, and all poets in town. How many homes does he visit, using these statistics? Draw a Venn diagram.

    Homes with:

Children	800
Grandmothers	250
Poets	60
Children and grandmothers	100
Children and poets	10
Grandmothers and poets	3
Grandmothers and poets and children	1

27. A politician claims that all Catholics and all homeowners in town will vote for him. How many votes does he claim, using these statistics? Draw a Venn diagram.

Catholics	400
Homeowners	1200
Catholic homeowners	300

For the next four problems, assume the first two statements are correct and make a diagram to represent the relationships. Then answer the questions.

28. All crabs are birds. Some birds are blue.

    a. Can you be certain that all crabs are blue?

    b. Can you be certain that some crabs are blue?

29. Some elephants are lions. All lions have three eyes.

    a. Can you be certain that all elephants have three eyes?

    b. Can you be certain that some elephants have three eyes?

30. All rifles are automobiles. No automobiles are machines.

    a. Can you be certain that some rifles are machines?

    b. Can you be certain that no rifles are machines?

31. Some paper is white. No white things are usable things.

    a. Can you be certain some paper is usable?

    b. Can you be certain that some paper is not usable?

32. Cross out the letter in the word *fireplace* which is two letters before the letter that precedes the *l*.

33. If the second half of the second name of our first president contains the second letter in *cheese*, circle the second word in this sentence. Otherwise, circle the first word in this sentence.

34. If deleting the first, third, fifth and seventh letters in the word *education* leaves four more consonants than vowels, circle the first comma in this sentence. Otherwise, circle the second comma.

35. Cross out the letter in the name Jonathan which is two letters after the letter that precedes the *h.*

36. If the fifth letter in the word *sanctuary* is the eighteenth letter in the alphabet, circle it. Otherwise, circle the letter (in the word) which is the fourteenth letter of the alphabet.

37. *8 9 7 5 3 9 9 2 4 6.* If the sum of the second and the sixth numbers is greater than 14, circle the third number, unless the sum of the second and the sixth numbers is greater than 15, in which case circle the number which is one-half the last number in the series.

38. *9 8 7 6 5 4 3 2 1.* Take the difference between the first number and the sixth number. Write the difference here ____ . Now take the difference between the fifth number and the seventh number. Write it here ____ . Finally, take the difference between these two differences and write it here ____ .

39. *8 2 7 5 6 4 5 3 4.* If the difference between the first number and the fifth number is greater than the difference between the first number and the sixth number circle the seven. Otherwise take the difference between the differences and circle it.

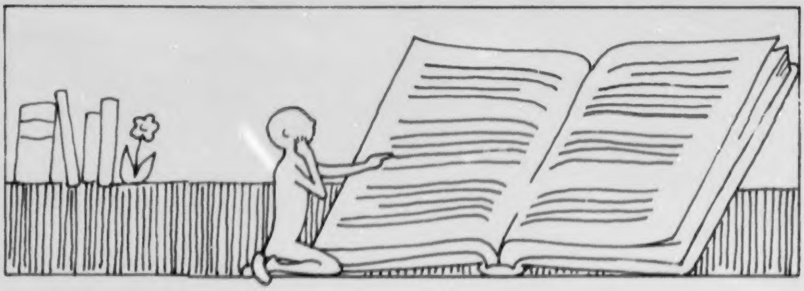
40. *1, 2, 4, 7, 4, 5, 3.* Subtract the third number from double the fifth number unless double the fourth number is more than the sum of the sixth and seventh numbers; in this case subtract the second number from double the sixth number. Then add two unless the third number is more than the sixth number in which case subtract one. Circle your answer below.

a. 9 *b.* 5 *c.* 10 *d.* 2 *e.* 7 *f.* 6 *g.* 1 *h.* 11

41. In a foreign language *ho lew gi* means "buy every dog," *lew ra* means "dog food," and *gi trj nk* means "every green car." Which words would you use to say "every food"? (Ignore the order of the words in answering the question.)

42. In a different language *si gumba lo* means "not very sweet," *ja lo* means "not brown," and *ba ja gumba* means "very brown coffee." How would you say "sweet coffee" in this language?

a. lo gumba *b. ja gumba* *c. ba si* *d. gumba ba* *e. ja si*



V. SIX MYTHS ABOUT READING

When you read textbooks and other technical material for full comprehension, you must read as carefully as you read the word problems in the last chapter. Occasionally you may not have time to read carefully and you may just skim the material. But you should recognize that when you skim you will not comprehend many of the details. You cannot learn mathematics by skimming a math text, nor can you learn chemistry, physics, biology or any other science by skimming textbooks in those areas.

There is a great deal of misinformation about reading. Here are six popular myths about reading which research has shown to be false.

Myth 1. Don't Subvocalize When You Read

You sometimes hear the advice that you should not move your lips, tongue or throat muscles when you read. You should not even hear the words in your mind when you read. You should be a totally "visual" reader. In some books, teachers have been advised to give young pupils candy or gum to prevent subvocalizing, and if necessary to even put a pencil or a ruler in a child's mouth to save him from this crippling habit.

A series of studies have now shown that subvocalizing is useful and perhaps even necessary for good comprehension of difficult material. For example, in one experiment college students who were taught to suppress their subvocalizing responses were only able to maintain good comprehension for easy reading material. Their comprehension of difficult reading material suffered drastically when they didn't subvocalize as they read.

All the evidence indicates that you should subvocalize freely when you read. It can produce better comprehension of technical material and a fuller appreciation of literary writing where alliteration and other poetic devices depend on hearing the words.

Myth 2. Read Only The Key Words

This advice is completely illogical. How can you know which words are the key words, until you first read the words? The advice assumes you have some magical, subliminal mechanism which allows you to pre-read the words and select out the key words which you will then read.

When students do try to read just key words they frequently emerge with a misinterpretation of the material. For example, one student I worked with read the following sentence silently:

> Some scraps of evidence bear out those who hold a very high opinion of the average level of culture among the Athenians of the Great Age.

I asked the student what the sentence said and he replied: "The level of Greek culture was very high." I said: "How about the first part of the sentence—some scraps of evidence?" He answered that he had skipped over that part and had tried to read just the key words, namely, "high . . . level of culture . . . Athenians."

Myth 3. Don't Be A Word-By-Word Reader

Emerald Dechant, a prominent reading researcher, made the following comments on this myth in the *Eleventh Yearbook of the National Reading Conference:*

> For years, and in many textbooks today, teachers have been and are being urged to teach the child to read two and three words per fixation. However, the best studies show that even college students rarely read more than one word per fixation. The assumption that children could, or at least normally would, recognize such large units, was based on misinterpretations of tachistoscopic research and resulted from a misunderstanding of the basic differences between tachistoscopic and normal reading. The limiting factor in recognition is the mind rather than the eye.[1]

Myth 4. Read In Thought Groups

This myth is closely related to myth 3. Since good readers basically read one word at a time, they obviously do not read in thought groups.

Naturally in reading you group words together mentally. Verbs and prepositions link nouns with other nouns, and so on. But you cannot "read in thought groups" in the sense of visually focussing on groups of words which form thoughts. In fact, this would be logically impossible. You couldn't know which words formed a "thought group" until you first read the words. It would be impossible to read by moving your eyes from one thought group to another.

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[1] Emerald Dechant, "Misinterpretations of Theory and/or Research Lead to Errors in Practice." In *Eleventh Yearbook of the National Reading Conference*, Emery P. Bliesmer and Ralph C. Staiger, editors, Milwaukee, WI: The National Reading Conference, Inc., 1962, page 127.

Myth 5. You Can Read At Speeds Of 1,000 Or More Words A Minute—Without Any Loss of Comprehension

Speed reading "experts" say if you read 250-300 words a minute you are plodding along at a horse-and-buggy rate and wasting your time. You should be reading three to ten times that fast. However, a sample of University of Michigan professors was found to read at an average rate of 303 words per minute, and the average rate for Harvard freshmen was 300 words per minute. Furthermore, a number of experiments have found that when people who have learned to skim at 600 words a minute or more cut back to 300 words per minute their comprehension improves. In study after study, approximately 300 words per minute has turned out to be the maximum rate at which people can read without sacrificing comprehension.

Students preparing for tests like the SAT, GRE, or LSAT can rest assured that a reading speed of 250-300 words a minute will allow them to attain a very high score if their comprehension is strong. Their preparation should consist mainly of strengthening their vocabulary and comprehension skills, not attempting to whip-up their reading speed.

Myth 6. Don't Regress Or Re-read

Speed reading "experts" say you should never regress or re-read a section of material, even if you feel you have not understood it well. Re-reading is said to be a bad reading habit and totally unproductive. Instead you should forge ahead and your understanding will be clarified as you read on.

Studies show that good readers do not follow this advice. With textbooks and other complicated material they must frequently re-read sentences and paragraphs to get the full meaning.

A Speed Reading Guarantee

One of the major speed reading companies supports its claim by offering to return a portion of your payment if they fail to triple your "reading efficiency." This sounds impressive to many people because they don't know what their "reading efficiency" represents. Here is the definition of reading efficiency:

Reading Efficiency = % Comprehension x Reading Speed

Assume you begin the speed reading course as a good reader who reads at 300 words per minute and answers 95% of the questions correctly on the comprehension test. Your reading efficiency is calculated like this:

Reading Efficiency = 95% x 300
= 285

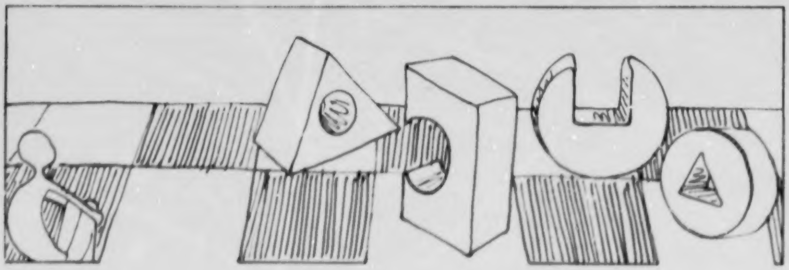
You then complete the speed reading course. Suppose on the test to determine your new reading efficiency you read 2,000 words a minute with 55% comprehension. Your new reading efficiency is calculated to be:

Reading Efficiency = 55% x 2,000
= 1,100

Your reading efficiency has more than tripled. But what good is a reading efficiency of 1,100 if it is based on a comprehension score of 55%? Would you want a surgeon to operate on you with anything less than his maximum possible comprehension of medical texts? Don't lawyers, nuclear engineers and auto mechanics require the maximum possible comprehension to do the best possible job?

The Reverend Jesse Jackson has been traveling around the country sponsoring a program called Push-Excel in which students and parents make a commitment for two to three hours of homework a night to improve school performance. Yet a major speed reading company has sponsored TV advertisements telling students they're missing the boat if they waste two or three hours reading textbooks every night. By taking the speed reading course, reading assignments can be completed in 20 to 30 minutes and students can spend more time out partying with their friends.

The vast majority of reading experiments indicate that Jesse Jackson is offering sounder advice than the speed reading advertisements. If you want to master your academic subjects and perform well on tests, you must read with care and thoroughness, and give the work the time it requires. There are no magical shortcuts. In reading textbooks and technical material, use all the activities of good problem solvers described in Chapter III.



VI. ANALOGIES

Section 1 Introduction

We often explain and emphasize our ideas by using analogies. For example, when we say "John is a chip off the old block," we bring home forcefully the similarity between John and his father. In effect we are saying John is so similar to his father, it's as if John were cut from the same material. Similarly, we read in the Bible "The Lord is my shepherd"—and from just these five words we get a clearer feeling for the protection and comfort felt by the Psalmist than we might from a lengthy, literal essay. Biologists tell us "the heart is like a mechanical pump," and we immediately understand the heart's role in circulating the blood. And in physics, new insights were gained when Lord Rutherford suggested that the solar system be used as a model for the atom, with electrons around the nucleus taken as analogous to planets around the sun.

Each of these statements or comparisons originates in *analogous* thinking. This is shown by presenting each of the ideas in the basic form of the simple analogy.

> John is similar to his father as a chip is similar to the block from which it is cut.
>
> The Lord cares for the Psalmist as a shepherd cares for his sheep.
>
> The heart moves the blood as a mechanical pump moves fluids.
>
> Electrons circle an atom's nucleus as planets circle the sun.

Analogies help us explain ideas to other people. More than that, they often lead us to new discoveries and inventions. Alexander Graham Bell believed that an apparatus for transmitting human conversation—a telephone—could be constructed. But he had enormous difficulty perfecting a means for converting voice sounds to electrical impulse so that speech could be carried through wires.

At one point he tried attaching a harp to an electromagnet. Striking a tuning fork in the vicinity of the harp caused selected strings to resonate, producing electrical waves. But this ponderous contraption was unsatisfactory for conversation.

A friend—the prominent Boston aurist, Dr. Clarence Blake—suggested that Bell study the human ear rather than continue experimenting with mechanical instruments, and gave the inventor an ear with connecting organs cut from the head of a cadaver. Bell observed that although the eardrum was extremely thin and light, it was able to move the three heavy inner-ear bones which formed the mechanical linkage of the ear. Bell later recalled, "it struck me that the bones of the human ear were very massive, indeed, as compared with the delicate thin membrane that operated them, and the thought occurred that if a membrane so delicate could move bones relatively so massive, why should not a thicker and stouter piece of membrane move my piece of steel."

That was the breakthrough. Various iron disks and membranes were tried. Finally in 1875 a diaphram of gold-beater's skin (which is very thin) was fastened to a metal armature adjacent to an electromagnet and the first working microphone, an integral part of any telephone, was created.

This bit of history is an example of a *biological analogy* furnishing critical information for a mechanical invention. The analogy can be expressed:

> Eardrum functions in the ear as diaphram functions in the telephone.

The strategy of looking for biological analogies to aid in technological developments is so fruitful that an entirely new science called Bionics has come into existence. The *Journal of Creative Behavior* describes Bionics in the following way:

> Bionics is a new approach to system design. It is the study of the structure, function, and mechanisms of plants and animals to gain design information for analogous man-made systems.[1]

This chapter asks you to work analogy problems. The primary role which analogous thinking plays in scientific invention, mathematical induction, and literary creation indicates that when a person systematically analyzes an analogy he uses the same mental skills that are important in comprehending and integrating all areas of advanced human knowledge. As you work the analogy problems in the chapter, you will find it necessary to spell ideas out fully, formulate precise relationships of facts, seek correspondences between diverse ideas, and compare relationships for similarities and differences. These are the activities which underlie the mastery of all academic courses, stretching from poetry to political science to calculus. These are the mental tools which man has used to shape all cultural and technological achievements.

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[1] "Bionics," *Journal of Creative Behavior.* Vol. 2, No. 1, Winter, 1967. P. 52-57.

Section 2 The "Relationship Sentence"

Here is an analogy:

Gills are related to *fish* as *lungs* are related to *humans.*

To really understand this analogy you must be able to show that the relationship between gills and fish is in some way the same as the relationship between lungs and humans. You must be able to spell out exactly what the relationship is.

Gills are used for breathing by *fish.*
Lungs are used for breathing by *humans.*

You could show the relationship with a "Relationship Sentence" like this:

Relationship Sentence: ________ are used for breathing by ________.

Section 3 Choosing Relationship Sentences

Each of the following problems presents an analogy and three possible relationship sentences. Your task is to pick the correct relationship sentence and substitute both pairs of words into the sentence to show it is correct.

Example 1

Carpenter is to *saw* as *plumber* is to *wrench.*

Relationship Sentence a. A ________ is a ________ .
Relationship Sentence b. A ________ cuts wood with a ________ .
Relationship Sentence c. A ________ uses a tool called a ________ .

Which relationship sentence is correct? Let's try substituting both pairs of words in relationship sentence a.

Relationship Sentence a. A *carpenter* is a *saw.*
A *plumber* is a *wrench.*

This is wrong. A carpenter is not a saw and a plumber is not a wrench.

Let's try relationship sentence b.

Relationship Sentence b. A *carpenter* cuts wood with a *saw.*
A *plumber* cuts wood with a *wrench.*

This is wrong. A plumber does not cut wood with a wrench.

Finally we'll try relationship sentence c.

Relationship Sentence c. A *carpenter* uses a tool called a *saw.*
A *plumber* uses a tool called a *wrench.*

This is a good relationship sentence. It shows why we say the relationship between carpenter and saw is similar to the relationship between plumber and wrench.

One more example is presented below.

Example 2

Choose the correct relationship sentence. Then copy it twice; once with stewardess-airplane substituted into the blanks and the second time with waitress-restaurant in the blanks.

Stewardess is to *airplane* as *waitress* is to *restaurant.*

Relationship Sentence a. A ______ gives safety instructions in a(n) ______ .
Relationship Sentence b. A ______ works in a(n) ______ .
Relationship Sentence c. A ______ is a(n) ______ .

Solution

The correct answer is relationship sentence b.

I. first word pair: A *stewardess* works in an *airplane.*
II. second word pair: A *waitress* works in a *restaurant.*

There is one complication. You say "an" airplane, but you say "a" restaurant. Because of this, the *n* is put in parenthesis in the relationship sentence.

A ______ works in a(n) ______ .

The parenthesis means that the *n* can be either used or omitted, whichever is appropriate.

ANALOGY PROBLEMS

For each of the following analogies, pick the best relationship sentence. Then copy that relationship sentence twice, once with the first word pair placed in the blanks, and once with the second word pair. Include or omit any letters in parentheses as appropriate.

Here is an example.

book is to *paper* as *shirt* is to *cloth*

a. A ________ is made of linen or some other ________ .

b. A ________ is words printed on ________ .

c. A ________ is primarily made of ________ .

Sentence c is correct.

I. first word pair: A *book* is primarily made of *paper.*

II. second word pair: A *shirt* is primarily made of *cloth.*

1. *guitar* is to *pick* as *fiddle* is to *bow*

a. A ________ is played with a ________ .

b. A ________ is plucked with a ________ .

c. A ________ is a ________ .

I. first word pair:

II. second word pair:

2. *water* is to *thirsty* as *food* is to *hungry*
   a. You drink ________ when you are ________ .
   b. You drink ________ and eat ________ .
   c. You consume ________ when you are ________ .

I. first word pair:

II. second word pair:

3. *bicycle is to airplane as cabin is to skyscraper*
   a. A ________ and a(n) ________ are used for travel.
   b. A ____ is simpler but used for the same purpose as a(n) ____ .
   c. You can live in a ________ or a(n) ________ .

I. first word pair:

II. second word pair:

4. *hotter* is to *hot* as *colder* is to *cold*
   a. ________ means the same as ________ .
   b. ________ means to ________ .
   c. ________ means more ________ .

I. first word pair:

II. second word pair:

5. *decrease* is to *smaller* as *magnify* is to *larger*

   a. ________ means ________ .

   b. ________ means make ________ .

   c. ________ means make shorter or ________ .

I. first word pair:

II. second word pair:

6. *student* is to *truant* as *soldier* is to *A.W.O.L.*

   a. A ________ out of school is ________ .

   b. A ________ can be court marshalled for being ________ .

   c. A ________ who is absent illegally is ________ .

I. first word pair:

II. second word pair:

7. *polluted* is to *pure* as *tainted* is to *undefiled*

   a. ________ and ________ start with the same letter.

   b. ________ and ________ are opposite in meaning.

   c. ________ and ________ mean dirty.

I. first word pair:

II. second word pair:

8. *dam* is to *flood* as *vaccination* is to *disease*
   a. A ________ holds back water to prevent a ________ .
   b. A ________ is given by a doctor to prevent a ________ .
   c. A ________ is used to prevent a ________ .

I. first word pair:

II. second word pair:

9. *fence* is to *garden* as *bumper* is to *car*
   a. A ________ helps protect a ________ .
   b. A ________ keeps trespassers out of a ________ .
   c. A ________ surrounds a ________ .

I. first word pair:

II. second word pair:

10. *gluttony* is to *obese* as *temperance* is to *trim*
   a. ________ results in a person being ________ .
   b. ________ and inactivity result in a person being ________ .
   c. ________ and exercise result in a person being ________ .

I. first word pair:

II. second word pair:

11. *20* is to *10* as *50* is to *25*

a. ________ is ten more than ________ .

b. ________ is twice ________ .

c. ________ is one-half of ________ .

I. first word pair:

II. second word pair:

12. *20* is to *10* as *50* is to *40*

a. ________ is ten more than ________ .

b. ________ is twice ________ .

c. ________ is one-half of ________ .

I. first word pair:

II. second word pair:

13. *50* is to *48* as *67* is to *64*

a. ________ is two more than ________ .

b. ________ is larger than ________ .

c. ________ is smaller than ________ .

I. first word pair:

II. second word pair:

14. *30* is to *20* as *60* is to *40*
    a. ________ is ten more than ________ .
    b. ________ is one and one-half times ________ .
    c. ________ is smaller than ________ .

I. first word pair:

II. second word pair:

15. *20* is to *40* as *30* is to *60*
    a. ________ is twice ________ .
    b. ________ is larger than ________ .
    c. ________ is one-half of ________ .

I. first word pair:

II. second word pair:

16. *40* is to *30* as *80* is to *60*
    a. ________ is three-quarters of ________ .
    b. ________ is one and one-third times ________ .
    c. ________ is smaller than ________ .

I. first word pair:

II. second word pair:

17. *50* is to *60* as *10* is to *12*

a. ________ is 10 less than ________ .

b. ________ is one and one-fifth times ________ .

c. ________ is five-sixths of ________ .

I. first word pair:

II. second word pair:

18. *70* is to *80* as *14* is to *16*

a. ________ is eight-sevenths of ________ .

b. ________ is seven-eighths of ________ .

c. ________ is seven-fourteenths of ________ .

I. first word pair:

II. second word pair:

19. *warm* is to *hot* as *smile* is to *laugh*

a. (A) ________ is a mild form of (a) ________ .

b. (A) ________ and a ________ stem from something funny.

c. (A) ________ and (a) ________ are the opposite of cold.

I. first word pair:

II. second word pair:

20. *roar* is to *sea* as *howl* is to *wind*

    a. ________ is the sound of water crashing in the ________ .

    b. ________ is the sound of the turbulent air of the ________ .

    c. ________ is the sound of the ________ .

I. first word pair:

II. second word pair:

21. *surgeon* is to *scalpel* as *writer* is to *words*

    a. The tool of the ________ is (the) ________ .

    b. A ________ performs appendectomies with a ________ .

    c. A ________ uses ________ to communicate ideas.

I. first word pair:

II. second word pair:

22. *menu* is to *restaurant* as *guidebook* is to *city*

    a. A ________ shows the foods available in a ________ .

    b. A ________ lists what is available in a ________ .

    c. A ________ lists the sights in a ________ .

I. first word pair:

II. second word pair:

23. *rake* is to *hoe* as *fork* is to *spoon*

   a. A ________ has prongs while a ________ is solid.
   b. A ________ and a ________ are used in gardening.
   c. A ________ and a ________ are eating utensils.

   I. first word pair:

   II. second word pair:

24. *aspirin* is to *headache* as *solace* is to *misfortune*

   a. ________ is taken with water to relieve a ________ .
   b. ________ relieves the pain of a ________ .
   c. ________ causes a ________ .

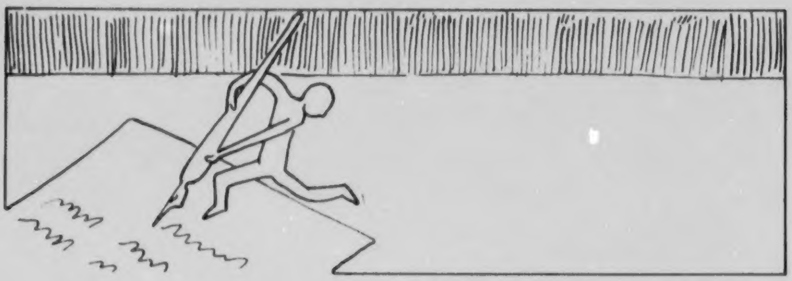
   I. first word pair:

   II. second word pair:

When you have completed this series of problems, discuss your answers in class to be sure they are correct.

Homework Assignment

After you have had an opportunity to work some problems in class, make up several problems of the same type for homework. Each problem should include an analogy and three possible relationship sentences, only one of which is correct. Also, show that both pairs of words can be placed into the correct relationship sentence.



VII. WRITING RELATIONSHIP SENTENCES

Section 1 Introduction

The problems in this chapter are very similar to those in the last chapter except that you must write the relationship sentences yourself. Here is an example.

paper is to *trees* as *iron* is to *ore*

A good problem solver who read and thought aloud as she attempted to write a relationship sentence for this analogy responded as follows: "Paper is to trees as iron is to ore. I think you get iron from ore. And I guess you also get paper from trees. So that's the analogy. Paper is produced from trees and iron is produced from ore."

The problem solver wrote her ideas in two separate sentences so that she could see them clearly and compare them. Then she wrote the relationship sentence with blanks.

Sentence 1. Paper is produced from trees.

Sentence 2. Iron is produced from ore.

Relationship Sentence: ________ is produced from ________ .

Section 2 Sample Problem

Formulate a relationship sentence which explains why the analogy below holds true. Follow the steps used by the problem solver in the last section.

First write two separate sentences—one sentence showing the relationship between the first pair of words in the analogy, and a second sentence showing the relationship between the second pair of words.

Then write the relationship sentence with blanks to accommodate either pair of words.

If you are using this book in a class, one student should solve the problem aloud and write the sentences on the chalkboard.

mouth is to *talk* as *hand* is to *grasp*

Sentence 1.

Sentence 2.

Relationship Sentence:

Section 3 Solution

Here is the analogy you were presented with in section 2.

mouth is to *talk* as *hand* is to *grasp*

In writing the relationship sentence, you may have begun by saying "A mouth can be used to talk, and a hand can be used to grasp."

A mouth can be used to talk.

A hand can be used to grasp.

Then you might have written the core of these sentences with blanks, allowing either word pair to be substituted correctly.

A _______ can be used to _______ .

The relationship sentence that you wrote may have been a little different than this one. For example, here are two equally good variations.

The _______ is used to _______ .

A person employs his _______ to _______ .

Any relationship sentence which shows why the analogy holds is fine.

In working the problems in this chapter, there is only one restriction to keep in mind. Write the relationship sentence so that the word pairs remain in the same order as they occur in the original analogy. For instance, consider the relationship sentence.

You can _______ with your _______ .

Notice that in order to place the word pairs (mouth-talk and hand-grasp) meaningfully into the blanks you must reverse their order. For example, you must change "mouth-talk" to "talk-mouth."

You can talk with your mouth.

You can grasp with your hand.

In working the problems in this chapter, avoid writing relationship sentences which invert word order. Instead, always write the relationship sentence in a way which allows you to place the words into the blanks in the same order that they occurred in the original analogy. This will make it easier to check whether the relationship sentence is totally correct. You will see how important this is in subsequent chapters when you begin analyzing more complex analogies.

Section 4 Examples

Relationship sentences are very much like mathematical equations. For example, consider this number analogy:

6 is to *2* as *21* is to *7*

A satisfactory relationship sentence is:

________ is three times ________ .

If you know a little algebra, you will recognize that this relationship sentence is equivalent to the following algebraic equation.

$y = 3x$

This is true because:

$$\begin{aligned} 6 &= 3(2) \\ 21 &= 3(7) \end{aligned}$$

A relationship sentence is written in common language (such as standard English)—and can show a connection either between words, or between numbers. On the other hand, an algebraic equation is written in mathematical symbols, and can only show a connection between numbers.

Let's look at a few more facts about number analogies. Consider this one:

70 is to *30* as *35* is to *15*

At first sight it might occur to a person that 70 is 40 more than 30, and he might consider writing this relationship sentence:

________ is 40 more than ________ .

But when he checks this against the second half of the analogy, he sees that 35 is not 40 more than 15. He would then discard his first conclusion, and work his way to the correct relationship:

________ is 7/3 of ________ .

By way of contrast, consider another analogy.

50 is to *10* as *110* is to *70*

In this case it is true that for both the first pair of numbers and the second pair of numbers, the difference is 40. With this analogy a good relationship sentence is:

________ is 40 more than ________ .

This relationship sentence can also be written as an algebraic equation:

$y = x + 40$

We check it by substituting the numbers from the analogy:

$$50 = 10 + 40$$
$$110 = 70 + 40$$

The close connection between relationship sentences and algebraic equations shows something very important. When you practice writing relationship sentences (with blanks into which either pair of words can be substituted) you are sharpening the same analytical skill that you depend on in understanding and manipulating all algebraic expressions. Strange as it may sound, by practicing verbal analogies you can increase your strength in mathematical problem solving.

Homework Assignment

Make up several analogies. For each analogy write one sentence showing the relationship between the first pair of words, another sentence showing the relationship between the second pair of words, and a relationship sentence.

RELATIONSHIP PROBLEMS

The following problems involve writing relationship sentences for analogies. Work in pairs, and do all your reading and thinking aloud. Also, to guide your thinking and make it more visible to your partner, please use this procedure:

1. Examine the analogy, reading and thinking aloud.
2. When you have discovered the common relationship, write two separate sentences: One sentence using the common relationship and the first pair of words, and another sentence using the common relationship and the second pair of words.
3. As your final step, write the relationship sentence with the blanks—and check that it is entirely correct.

Here is an example:

legs are to *chair* as *wheels* are to *car*

Sentence 1. Legs support a *chair.*

Sentence 2. Wheels support a *car.*

Relationship Sentence: _______ support a _______ .

Problems

1. *arrive* is to *depart* as *find* is to *lose*

Sentence 1.

Sentence 2.

Relationship Sentence:

2. *books* are to *library* as *paintings* are to *museum*

Sentence 1.

Sentence 2.

Relationship Sentence:

3. *pen* is to *typewriter* as *horse* is to *automobile*

   Sentence 1.

   Sentence 2.

   Relationship Sentence:

4. *key* is to *typewriter* as *steering wheel* is to *automobile*

   Sentence 1.

   Sentence 2.

   Relationship Sentence:

5. *author* is to *book* as *mother* is to *child*

   Sentence 1.

   Sentence 2.

   Relationship Sentence:

6. *electricity* is to *motor* as *ambition* is to *human*

   Sentence 1.

   Sentence 2.

   Relationship Sentence:

7. *artist* is to *talent* as *athlete* is to *coordination*

   Sentence 1.

   Sentence 2.

   Relationship Sentence:

8. *roots* are to *plant* as *mouth* is to *animal*

   Sentence 1.

   Sentence 2.

   Relationship Sentence:

9. *hand* is to *shoulder* as *foot* is to *hip*

   Sentence 1.

   Sentence 2.

   Relationship Sentence:

10. *peacock* is to *bird* as *tuxedo* is to *suit*

    Sentence 1.

    Sentence 2.

    Relationship Sentence:

11. *verdict* is to *jury* as *sentence* is to *judge*

    Sentence 1.

    Sentence 2.

    Relationship Sentence:

12. *hound* is to *fox* as *lion* is to *zebra*

    Sentence 1.

    Sentence 2.

    Relationship Sentence:

Note: For the next five problems, do not use the relationship sentence "________ is smaller/larger than ________ ." Instead, write one which describes the relationship more fully, such as "________ is 25 more than ________ ."

13. *30* is to *10* as *45* is to *15*

    Sentence 1.

    Sentence 2.

    Relationship Sentence:

14. *10* is to *40* as *80* is to *110*

    Sentence 1.

    Sentence 2.

    Relationship Sentence:

15. *10* is to *30* as *40* is to *120*

Sentence 1.

Sentence 2.

Relationship Sentence:

16. *50* is to *20* as *90* is to *60*

Sentence 1.

Sentence 2.

Relationship Sentence:

17. *50* is to *20* as *25* is to *10*

Sentence 1.

Sentence 2.

Relationship Sentence:

18. *roots* are to *plant* as *foundation* is to *building*

Sentence 1.

Sentence 2.

Relationship Sentence:

19. *earth* is to *sun* as *moon* is to *earth*

Sentence 1.

Sentence 2.

Relationship Sentence:

20. *fishes* are to *school* as *wolves* are to *pack*

Sentence 1.

Sentence 2.

Relationship Sentence:

21. *lamp* is to *light* as *furnace* is to *warmth*

Sentence 1.

Sentence 2.

Relationship Sentence:

22. *warm* is to *hot* as *cool* is to *cold*

Sentence 1.

Sentence 2.

Relationship Sentence:

23. *always* is to *often* as *never* is to *seldom*

Sentence 1.

Sentence 2.

Relationship Sentence:

24. *antidote* is to *poison* as *teacher* is to *ignorance*

Sentence 1.

Sentence 2.

Relationship Sentence:

25. *cube* is to *square* as *cylinder* is to *circle*

Sentence 1.

Sentence 2.

Relationship Sentence:

26. *threat* is to *punch* as *growl* is to *bite*

Sentence 1.

Sentence 2.

Relationship Sentence:

27. *itinerary* is to *trip* as *table of contents* is to *book*

Sentence 1.

Sentence 2.

Relationship Sentence:

28. *horns* are to *gore* as *bombs* are to *level*

Sentence 1.

Sentence 2.

Relationship Sentence:

29. *peninsula* is to *ocean* as *bay* is to *land*

Sentence 1.

Sentence 2.

Relationship Sentence:

30. *idea* is to *action* as *seed* is to *flower*

Sentence 1.

Sentence 2.

Relationship Sentence:

31. *sun* is to *moon* as *bulb* is to *reflector*

Sentence 1.

Sentence 2.

Relationship Sentence:

32. *depressed* is to *economy* as *incapacitated* is to *person*

Sentence 1.

Sentence 2.

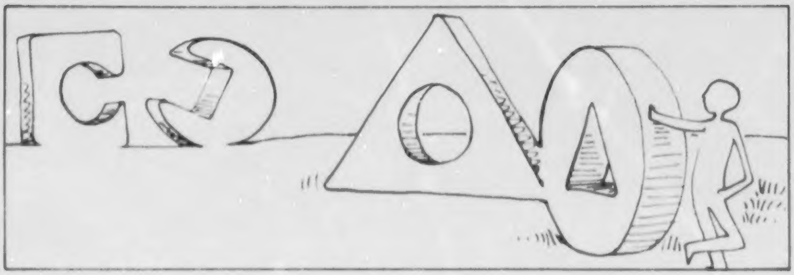
Relationship Sentence:

33. *bi* is to *two* as *pent* is to *five*

Sentence 1.

Sentence 2.

Relationship Sentence:



VIII. HOW TO FORM ANALOGIES

Section 1 Introduction

This chapter asks that you solve analogy problems of the type that are included in most IQ tests, job applicant tests, and college-entrance exams such as the College Boards, Gradute Record Exam, and Miller's Analogies Test. Working these problems will help you score higher on such tests. In addition, it will strengthen your analytical skills for all academic work.

Here is a sample problem. Your task is to select the answer which forms the best analogy when the words are placed into the blank. Please work the problem aloud. Read the entire problem aloud, including all the answer choices. For each of the answer choices, explain why it is either correct or incorrect—so the rest of the class understands your reasons for picking one alternative as the best answer.

After you have chosen your answer, write a relationship sentence which shows the rationale underlying the analogy.

thermometer is to *temperature* as ______________ is to ______________ .

a. telescope : astronomy
b. clock : minutes
c. scale : weight
d. microscope : biologist

Relationship Sentence:

Section 2 Problem Solver's Response

Here is how a good problem solver worked the problem presented in section 1. Read the solution aloud.

Original Problem

thermometer is to temperature as ______________ is to ______________ .

a. telescope : astronomy	*c.* scale : weight
b. clock : minutes	*d.* microscope : biologist

Problem Solver's Response

The Problem Solver read the relationship sentence, then the four options aloud, considering each one in turn.

"Thermometer is to temperature as *blank* is to *blank*." A thermometer measures temperature.

"Telescope : astronomy." A telescope doesn't measure astronomy.

"Clock : minutes." A clock measures minutes. So this might be a good answer.

"Scale : weight." A scale measures weight. This could also be the answer.

"Microscope : biologist." A microscope doesn't measure a biologist.

It seems as though there are two good answers. Let me go back over them. A thermometer measures temperature, a clock measures minutes, and a scale measures weight. The word "minutes" is different from "temperature" or "weight." A clock measures time, not minutes. Minutes are units of time—just as degrees are units of temperature, and pounds are units of weight.

So the best answer is *c*, scale : weight. Temperature and weight are dimensions. But minutes aren't dimensions; they are units of a dimension. A thermometer measures the dimension temperature, and a scale measures the dimension weight.

Section 3 Sample Problem

In the problem solution in the last section, the problem solver defined the relationship between the first pair of words (he observed that a thermometer measures temperature) and then looked to see if he could find a similar relationship in any of the other word pairs. Use the same approach with this problem. State the relationship which you see in the first pair of words. Then read each of the answer choices aloud and explain why the words do or do not have the same relationship.

horse is to *animal* as ____________________ is to ____________________.

a. cow : milk
b. farm : pig
c. oak : wood
d. saddle : stallion

Relationship Sentence:

Section 4 Problem Solver's Response

Here is how a good problem solver worked the problem presented in Section 3. Read the solution aloud.

Original Problem

horse is to *animal* as ________________ is to ________________ .

a. cow : milk
b. farm : pig
c. oak : wood
d. saddle : stallion

Problem Solver's Response

The Problem Solver read the relationship sentence aloud, then considered the four options in turn.

"*Horse* is to *animal* as blank is to blank." A horse is an animal.
It is a type of animal.

"*Cow : milk.*" A cow isn't a type of milk . . . it gives milk.

"*Farm : pig.*" A farm isn't a type of pig.

"*Oak : wood.*" Oak is a type of wood. So this forms an analogy.

"*Saddle : stallion.*" A saddle isn't a type of stallion.

Answer *c* is best. A horse is a type of animal and oak is a type of wood.

Section 5 Sample Problem

A common error in analogy problems is to select an answer which reverses the direction of the relationship. Be careful not to do that in the following problem.

State the relationship between the first two numbers, and then explain why each of the answer choices does or doesn't form an analogy.

2 is to *6* as ____________________ is to ____________________ .

a. 6 : 2
b. 3 : 1
c. 12 : 36
d. 12 : 60

Relationship Sentence:

Section 6 Sample Problem

In this problem it is easy to make the error of picking an answer which reverses the relationship between the words. State the relationship between the first pair of words. Then read each of the answer choices aloud and explain whether or not it fits the same relationship. Don't skip any of the answer choices. Read and discuss all four of them.

city is to *mayor* as ____________________ is to ____________________ .

a. president : country
b. government : business
c. senate : congress
d. business : manager

Relationship Sentence:

Section 7 Sample Problem

This is a difficult question which requires subtle reasoning. Use the same procedure you did with the previous problems. State the relationship between the first pair of words. Then explain fully why each of the answers is either correct or incorrect.

pack is to *wolves* as ________________ is to ________________ .

a. wheel : spokes
b. garage : cars
c. alphabet : letters
d. aquarium : fish

Relationship Sentence:

Section 8 Sample Problem

The following problem has a slightly different format, although the idea is the same. The task is to pick the pair of words that form the best analogy when placed in the blanks.

Please do all your reading and thinking aloud. Substitute the first pair of words into the blanks, state any relationship which you see, and explain why the answer does or doesn't form a good analogy. Do this for all four answer choices. Don't skip any of them.

____________________ is to *dollar* as *year* is to ____________________ .

a. money : calendar
b. dime : month
c. penny : century
d. savings : century

Relationship Sentence:

Section 9 Problem Solution

Original Problem

__________________ is to *dollar* as *year* is to __________________.

a. money : calendar
b. dime : month
c. penny : century
d. savings : century

Problem Solution

__________________ is to *dollar* as *year* is to __________________.

Money : calendar. Money is to dollar as year is to calendar. A dollar is an amount of money. But a calendar isn't an amount of year. So this doesn't seem like a good analogy.

Dime : month. Dime is to dollar—a dime is one-tenth of a dollar—as year is to month. A year is not part of a month.

Penny : century. Penny is to dollar—a penny is 1/100 of a dollar—as year is to century. A year is 1/100 of a century. So this is a good analogy.

Savings : century. Savings is to dollar as year is to century. A year is part of a century, but savings aren't generally part of a dollar. This doesn't seem to form an analogy.

The best answer is c, penny : century.

Relationship Sentence: A ______________ is 1/100 of a ______________.

Section 10 Sample Problem

Here is another problem with the new format. Read each of the answer alternatives aloud, substitute the words into the blanks, and explain any relationships that you see. Do this for all four of the answer choices, just as the problem solver did in the last problem solution.

__________________ is to *liquid* as *ice* is to __________________ .

a. flowing : solid
b. warm : cold
c. water : solid
d. milk : cream

Relationship Sentence:

Section 11 Sample Problem

Here is still one more format used with analogy problems. Again, the task is to select the answer which forms the best analogy.

Read each of the answer choices aloud, substitute the words into the blanks, state any relationships you see, and explain why an analogy is or is not formed.

tar is to __________________ as *coal* is to __________________ .

a. roofing : shovel
b. derived : heating
c. construction : heating
d. black : heating

Relationship Sentence:

Section 12 Problem Solution

Original Problem

tar is to ______________________ as *coal* is to ______________________ .

a. roofing : shovel
b. derived : heating
c. construction : heating
d. black : heating

Problem Solution

Tar is to ______________________ as *coal* is to ______________________ .

Roofing : shovel. Tar is to roofing as coal is to shovel. Tar is used in roofing, but it doesn't make any sense to say coal is used in shovel.

Derived : heating. Tar is to derived as coal is to heating. Coal is used in heating, but it doesn't make sense to say tar is used in derived.

Construction : heating. Tar is used in construction and coal is used in heating. So this forms an analogy.

Black : heating. Tar is black, but you don't say coal is heating.

The best answer is *c*, construction : heating.

Relationship Sentence: ________________ is used for ________________ .

Section 13 Sample Problem

Tests sometimes have analogy problems with several answers that form good analogies, but one answer that forms the best analogy. In solving such a problem, try to formulate a relationship sentence which excludes all but one answer, and explains why that answer is best. A good relationship sentence takes the haziness out of deciding which of several good answers is the best one.

Pick the best answer for this problem.

fur is to *bear* as ____________________ is to ____________________.

a. coat : man
b. warmth: animal
c. rug : floor
d. wool : sheep

Relationship Sentence:

Why are the following relationship sentences unsatisfactory?

(A) ____________________ keeps a ____________________ warm.

(A) ______________ is a protective covering of a ______________.

Section 14 Sample Problem

Pick the best answer for this problem. Use a dictionary if necessary.

centaur is to *horse* as ____________________ is to ____________________ .

a. woman : mermaid
b. mermaid : woman
c. mermaid : fish
d. fish : mermaid

Relationship Sentence:

Discussion Question

Why is this relationship sentence unsatisfactory?

A ________________________ is part ________________________ .

Section 15 Sample Problem

Some of the problems in this chapter are quite difficult. Finding the correct answer and the relationship common to the pairs will require an appreciable amount of search and analysis. A good way to work is as follows:

1. Begin with the first answer choice. Try putting the words into the blanks. See if you can find any relationship for the first word pairs. Also, see if you can find any relationship for the second word pairs. If both relationships are clear but not the same, then you can be pretty sure that the answer is incorrect. However, if the relationships are not clear, keep this in mind—and perhaps come back to the answer choice later.
2. Do this for each of the answer choices, narrowing down the possibilities.
3. Keep searching until you find an answer which gives you the same relationship for the first half and the second half of the analogy. Do not settle for an answer which is only partially correct. Keep digging until you find one which is totally correct.
4. Carry out your search by asking yourself how the words in each half of the analogy are the same, how they are different, and how they are related.

Here is a difficult problem. If you cannot find the best answer, go back over the steps listed above to be sure you have carried them all out.

______________________ is to *cave* as *car* is to ______________________.

a. stone : steel
b. primitive : modern
c. apartment house : horse
d. modern : primitive

Relationship Sentence:

Section 16 Problem Solution

Original Problem

______________________ is to *cave* as *car* is to ______________________ .

a. stone : steel
b. primitive : modern
c. apartment house : horse
d. modern : primitive

Problem Solution

______________________ is to *cave* as *car* is to ______________________ .

Stone : steel. Stone is to cave as car is to steel. Stone is what a cave is made of. But car is not what steel is made of. The words would have to be reversed to form an analogy.

Primitive : modern. Primitive is to cave as car is to modern. Primitive is what a cave is. But car isn't what a modern is. The words would have to be reversed to form an analogy.

Apartment house : horse. Apartment house is to cave as car is to horse. The relationships aren't immediately evident here. Perhaps this one should be looked at again later.

Modern : primitive. Modern is to cave as car is to primitive. This answer also seems wrong because the words are reversed. The word *modern* comes before the word *cave.* But the word *primitive* comes after the word *car.*

Answers *a, b* and *d* seem to be definitely wrong. Answer *c* must be looked at more closely.

Apartment house is to cave as car is to horse. What is the relationship between an apartment house and a cave? There is no obvious relationship.

What is the relationship between a car and a horse? A car is a means of transportation. A horse can also be used for transportation.

How about the relationship between an apartment house and a cave? An apartment house is a place to live. Can a cave be lived in? Yes, cavemen lived in caves, and some animals live in caves. So an apartment house and a cave are similar in that they are both things that can be lived in.

How is an apartment house different from a cave? An apartment house is a modern residence, whereas a cave is primitive.

Now the question is, can this be tied in with the relationship between a car and a horse?

A car and a horse can both be used for transportation. However, a car is a modern means of transportation, while a horse is a more primitive means of transportation.

Therefore, there is a common relationship between the pairs of words. You might say: An apartment house is the modern counterpart of a cave, and a car is the modern counterpart of a horse.

ANALOGY PROBLEMS

For the remaining problems in this chapter your instructor may ask you to work in pairs, taking turns as problem solver and listener. Please remember to read each answer alternative aloud and explain to your partner why you regard it as correct or incorrect. Don't dismiss any answer alternative as incorrect without offering some explanation to your partner. If you do, the communication process with your partner breaks down.

There are a total of 24 problems. After you have worked the first 12 problems, the class will stop and discuss the answers and relationship sentences. Make sure that you have a good reason for each answer, so that you can defend it as correct during the class discussion.

1. *horse* is to *hoof* as _______ is to _______ .

a. man : name
b. man : foot
c. man : head
d. animal : run

Relationship Sentence:

2. _______ is to *tennis* as _______ is to *hockey.*

a. racket : puck
b. net : ice
c. racket : stick
d. player : participant

Relationship Sentence:

3. _______ is to *plantation* as *car* is to _______ .

a. Kentucky : Detroit
b. tractor : passenger
c. agriculture : industry
d. tobacco : factory

Relationship Sentence:

4. ________ is to *time* as *foot* is to ________ .

a. late : long
b. clock : shoe
c. minute : distance
d. space : leg

Relationship Sentence:

5. ________ is to *June* as *July* is to ________ .

a. January : February
b. March : September
c. warm : hot
d. March : October

Relationship Sentence:

6. ________ is to *year* as *letters* are to ________ .

a. months : mailbox
b. months : number
c. months : alphabet
d. 1974 : A B C

Relationship Sentence:

7. *erosion* is to *soil* as ________ is to ________ .

a. farmland : desert
b. exhaustion : work
c. polluted : water
d. demoralization : character

Relationship Sentence:

8. ________ is to *cow* as *mutton* is to ________ .

a. bull : sheep
b. calf : sheep
c. beef : pork
d. beef : sheep

Relationship Sentence:

9. ________ is to *torso* as *branch* is to ________ .

a. arm : leaf
b. leg : twigs
c. fingernails : acorns
d. arm : trunk

Relationship Sentence:

10. ________ is to *hour* as *hour* is to ________ .

a. minute : second
b. minute : day
c. clock : minute
d. minute : long

Relationship Sentence:

11. ________ is to *whiskey* as *refinery* is to ________ .

a. distillery : gasoline
b. distillery : clothing
c. bottle : barrel
d. Tennessee : Texas

Relationship Sentence:

12. ________ is to *operation* as *writer* is to ________ .

a. plan : outline
b. hospital : library
c. surgeon : book
d. medicine : grammar

Relationship Sentence:

13. ________ is to *time* as *ruler* is to ________ .

a. minute : inch
b. clock : inch
c. clock : length
d. clock : straight

Relationship Sentence:

14. ________ is to *exercise* as *salary* is to ________ .

a. running : money
b. fitness : money
c. handball : work
d. fitness : work

Relationship Sentence:

15. ________ is to *carbohydrate* as *butcher* is to ________ .

a. baker : meat
b. baker : protein
c. calories : protein
d. potato : protein

Relationship Sentence:

16. *hospital* is to *disease* as ________ is to ________ .

a. lawyer : client
b. license bureau : license
c. police department : crime
d. United Nations : peace

Relationship Sentence:

17. ________ is to *right* as *weather* is to ________ .

a. wrong : climate
b. left : rain
c. write : whether
d. correct : report

Relationship Sentence:

18. *dress* is to *wool* as _______ is to _______ .

a. animal : dog
b. suit : jacket
c. door : glass
d. concrete : building

Relationship Sentence:

19. *suit* is to *jacket* as _______ is to _______ .

a. animal : head
b. dress : gown
c. animal : cat
d. car : Buick

Relationship Sentence:

20. _______ is to *grass* as _______ is to *sales*

a. homeowner : customer
b. fertilizer : advertising
c. seed : recession
d. weed killer : recession

Relationship Sentence:

21. *whale* is to *dolphin* as _______ is to _______ .

a. gorilla : chimpanzee
b. fish : mammal
c. smart : dumb
d. lungs : gills

Relationship Sentence:

22. _______ is to *action* as *seed* is to _______ .

a. idea : flower
b. idea : performance
c. carryout : grow
d. act : flower

Relationship Sentence:

23. _______ is to *ocean* as *lake* is to _______ .

a. ship : boat
b. island : land
c. Atlantic : Erie
d. rough : calm

Relationship Sentence:

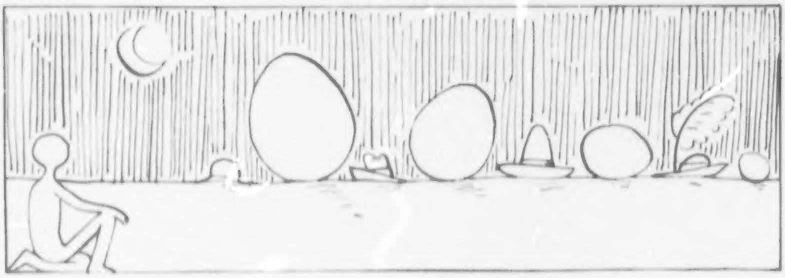
24. *Miami* is to *city* as _______ is to _______ .

a. state : Florida
b. Lincoln : president
c. city : state
d. city : south

Relationship Sentence:

Homework Assignment

Make up several analogy problems of the type dealt with in this chapter. For each of them write a problem solution which shows why three answers are incorrect and one is correct.



IX. ANALYSIS OF TRENDS AND PATTERNS

Section 1 Introduction

Patterns and trends are found very frequently in the physical and social sciences, as well as in mathematics. When they occur it is often useful to identify them precisely because they pave the way for predictions about future events. For example, here is the pattern of average minimum temperatures for certain months in New York City.

Year	1973				1974				1975			
Month	Jan.	Apr.	July	Oct.	Jan.	Apr.	July	Oct.	Jan.	Apr.	July	Oct.
Temp. Fahr.	30	47	66	49	29	44	68	46	32	40	68	53

Based on this pattern, approximately what would you expect the average minimum temperatures to be for January, April, July, and October of 1976?

Patterns and trends are found in almost all areas where regular, periodic observation is made. This includes changes in the distances between heavenly bodies; sales records of products ranging from shoes to automobiles; rainfall, wind velocity and other geological measures; and even human statistics such as birth, suicide and health patterns. Generally the cycles are more complex and erratic than the simple example above. Nevertheless, by systematically analyzing groups of observations, consistencies are often found which help bring organization to large bodies of facts, and furnish more comprehensible, usable pictures of the universe in which we live.

Section 2 Sample Problem

The problems in this chapter give you practice in identifying patterns and trends among numbers and letters. Not only will your grasp of patterns increase, but because patterns are really recurring relationships, working the problems will also improve your overall skill in analyzing relationships. Furthermore, many students have reported that by going through the problems in this chapter they developed greater confidence and skill in performing arithmetic operations.

Here is a sample problem.

> This series of letters follows a certain pattern. Try to discover the pattern and write the 3 letters which should come next.
>
> If you are using this book in a class, one student should solve the problem aloud at the chalkboard.
>
> *A B A C A D A E* ___ ___ ___
>
> There is one more part to this problem. Describe in your own words the pattern of the letters. You may find this a little difficult at first. However, if you were able to decide which 3 letters came next, then you have discovered the pattern.
>
> Write the pattern description below (or on the chalkboard).

Pattern description:

Section 3 Problem Solution

Original Problem

A B A C A D A E ___ ___ ___

Problem Solution

Problem Solver reads the series, pointing at the letters with her pen, and thinking aloud.	*A B* . . . *A C* . . . *A D.* The series is repeating *A*'s with letters in between that are in alphabetical order. The next one is *E* which is also this pattern. So *A F* and *A G* should follow.
Problem Solver filled the three spaces.	*A B A C A D A E* _*A*_ _*F*_ _*A*_
Problem Solver wrote the pattern description.	Pattern description: The letter *A* alternates with letters going up the alphabet.

Section 4 Sample Problem

In this problem numbers are arranged according to a pattern. Identify the pattern, decide which three numbers should come next, and write the pattern description. In a class one student should work the problem aloud at the chalkboard.

3 4 6 7 9 10 12 13 15 16 __ __ __

Pattern description:

Section 5 Problem Solution

Original Problem

3 4 6 7 9 10 12 13 15 16 __ __ __

Problem Solution

Problem Solver read and thought aloud, pointing to the numbers with his pen as he read them.

3 4 6 7 9 10 . . . 12 13. *3* to *4* is up 1. *6* to *7* is up 1. *9* to *10* is up 1. *4* to *6* is up 2. *7* to *9* is up 2. *10* to 12 is up 2.

Problem Solver wrote these differences above the problem as he computed them.

+1 +2 +1   +2 +1 +2
3 4 6 7 9 10 12 13 15 16 __ __ __

It looks like the series goes up 1, up 2, up 1, up 2. Let me check the rest.

12 to *13*—up 1. *13* to *15*—up 2. *15* to *16*—up 1.

Problem Solver filled the blanks as he computed each answer.

I'll fill in the blanks. The last one was *15* to *16* which was up 1. So next should be up 2 above 16. That would be *18*. Then up 1 would be *19*. Then up 2 would be *21*.

+1 +2 +1 +2 +1 +2 +1 +2 +1 +2
3 4 6 7 9 10 12 13 15 16 18 19 21

Problem Solver wrote the pattern description.

Pattern description: The pattern is add 1, add 2, add 1, add 2, etc.

Section 6 Sample Problem

Decide which 3 numbers should come next in this series and write the pattern description. In a class one student should work the problem aloud at the chalkboard.

2 7 4 9 6 11 8 13 __ __ __

Pattern description:

Section 7 Problem Solution

Original Problem

2 7 4 9 6 11 8 13 ___ ___ ___

Problem Solution

The Problem Solver read and thought aloud, pointing to the numbers with his pen.

2 7 4 9 6. The numbers seem to be going up and down. Let's see the rest. *11 8 13.* Yes, they're going up and down.

I'll look at the differences between the numbers to see if there is a pattern.

2 to *7* is up 5. *7* to *4* is down 3. *4* to *9* is up 5. *9* to *6* is down 3. *6* to *11* is up 5.

The Problem Solver wrote each of these differences as he computed it.

+5 −3 +5 −3 +5 −3 +5
2 7 4 9 6 11 8 13 10 15 ___ ___ ___

It seems to be going up 5, down 3, up 5, down 3. I'll check the rest.

11 to *8* is down 3. *8* to *13* is up 5.

I'll fill in the blanks. The last pair of numbers were *8* to *13,* which is up 5. So the next should go down 3. *13* minus *3* is *10.* I'll write that in the first blank.

+5 −3 +5 −3 +5 −3 +5
2 7 4 9 6 11 8 13 10 15 ___ ___

Next the numbers should go up 5. *10* plus *5* is *15.* I'll write that.

+5 −3 +5 −3 +5 −3 +5
2 7 4 9 6 11 8 13 10 15 ___

Then they should go down 3. *15* minus *3* is *12.*

+5 −3 +5 −3 +5 −3 +5
2 7 4 9 6 11 8 13 10 15

Pattern description: The pattern is add 5, subtract 3, add 5, subtract 3, etc.

Section 8 Writing The Pattern Description

In the last section the problem solver wrote this pattern description:

Pattern description: *Add 5, subtract 3, add 5, subtract 3, etc.*

There are various other ways to phrase this same idea. For example, here is a second way.

Pattern description: *Alternately add 5 and subtract 3.*

Any phraseology which fully expresses this notion is equally good. All that's important is that the pattern description show the basic principle underlying the pattern.

Sometimes different problem solvers will analyze the same pattern in different ways. Here is another way to approach the problem. Notice the numbers that are underlined.

$$\underline{2}\quad 7\quad \underline{4}\quad 9\quad \underline{6}\quad 11\quad \underline{8}\quad 13\quad \_\_\ \_\_\ \_\_$$

These numbers form the series: *2 4 6 8*. This is a series which simply increases by two each time.

Now look at the remaining numbers.

$$2\quad \underline{7}\quad 4\quad \underline{9}\quad 6\quad \underline{11}\quad 8\quad \underline{13}\quad \_\_\ \_\_\ \_\_$$

These numbers form the series: *7 9 11 13*. This series also increases by two each time.

A person could look at the original problem as two separate, alternating series—one starting with *2*, the other starting with 7—and both increasing by two. From this point of view a good pattern description would be:

Pattern description: *Two alternating series of numbers, each increasing by two.*

If in working the problems in this chapter, you and your partner come up with different pattern descriptions, first check that they are both really correct. If they are, you will see that they are actually two ways of looking at the same pattern, and both ways will lead to the same answers in filling the blanks.

For some of the problems which you will work later in the chapter, phrasing a good pattern description is the toughest challenge your verbalizing skills will face in this entire book. Nevertheless, challenging these speech and writing skills—exercising and strengthening them—is highly worthwhile. The talent to paint in words things that are seen and felt is the distinction of successful writers and public speakers. Teachers are also greatly dependent on this skill if they are to be effective. For example, an excellent gymnastics teacher is able to articulate clearly the positions, the movements, the twists and turns through which a student must direct his body. And you have seen in earlier chapters how important vocalizing is in teaching verbal and mathematical problem solving. Teaching in any area means communicating. In fact, for the rest of your life, in both professional and social settings, you will be trying to explain things to people. The better you do it, the greater your chances are for vocational advancement and personal happiness.

For the problems in this chapter, if you can fill the blanks then you know what the pattern is. If the pattern is somewhat complex, then don't expect to be able to describe it in just five or ten words. It may take 25 or 30 words—maybe 3 sentences—to describe the pattern fully enough that someone else could understand it from your description. Take all the time and space necessary to do the job of describing the pattern well.

Section 9 Sample Problem

Write the next 3 entries in this series and the pattern description. In a class, one student should work this problem aloud at the chalkboard.

1 z 3 w 9 t 27 q 81 __ __ __

Pattern description:

Section 10 Problem Solution

In reading this problem analysis take special note of three points which may help you in your own problem solving:

1. *One hypothesis considered, checked, rejected, and another one formulated.* At first the problem solver thought the letters and numbers were related. But after pursuing this for a while and not finding any relationship, she decided to deal with the letters and the numbers separately.

2. *Confusion, error, checking, writing the alphabet, and correction.* In analyzing the letter pattern, the problem solver got confused and made an error. However, good problem solvers continually re-check work that they are in any way unsure about. In re-checking, the problem solver wrote out the alphabet rather than trying to analyze the pattern completely in her own mind. She knew this would lead to greater accuracy, and it allowed her to find and correct her mistake.

3. *Blanks for letters filled first.* The problem solver filled the blanks with the appropriate letters before starting to analyze the numbers. She didn't depend on her memory to recall the letters later. Instead, she wrote them as soon as she figured them out.

Original Problem

1 z 3 w 9 t 27 q 81 __ __ __

Problem Solution

Problem Solver read and thought aloud.

1 z 3 w 9 t 27 q 81. The problem has numbers alternating with letters. Let me see if the numbers correspond to the letters in some way. *1 z.* *Z* is the last letter of the alphabet and 1 is the first number. Maybe the numbers are in reverse order of the letters. *3 w.* Let's see. *s t u v w x y z.* *W* is not the third letter from the end of the alphabet. The final entries are *q 81.* There aren't 81 letters in the alphabet, so I don't see any relationship between the numbers and the letters. Let me look at just the letters.

z w t q. These seem to be going backwards in the alphabet. I'll count the letters off. *q r s t.* That's 4 letters. *u v w*—that's 3. *w x y z*—that's 4 again.

Problem Solver counted on her fingers and wrote the answers above the problem.

4 3 4
1 z 3 w 9 t 27 q 81 __ __ __

Let me check this. I better write out part of the alphabet.

l m n o p q r s t u v w x y z

q . . . r s . . . t. So there are 2 letters in between *q* and *t.* *t . . . u v . . . w.* There are 2 letters in between *t* and *w.* *w x y z.* There are 2 letters between *w* and *z.* I'll correct what I wrote.

2 2 2
1 z 3 w 9 t 27 q 81 __ __ __

I wonder what I did wrong before. I guess I was saying that in going from *q* to *t,* or from *w* to *z,* there are 4 letters if you include where you start and where you end. But I made a mistake in going from *t* to *w.* I forgot to count the first letter, so I only got 3. Anyway, I'll leave it with the 2's. It means that there are 2 letters in between each letter in the series.

The numbers and letters alternate. Let me fill in the letters first. The last entry is a number, so the first blank has to be a letter, then the next one is a number, and the last one is a letter. So I need 2 letters. I'll look at the part of the alphabet that I wrote before. From *q,* if I skip back 2 letters I'll be at *n.* I'll write that in the first blank.

2 2 2
1 z 3 w 9 t 27 q 81 n __ __

Now I have to skip over 2 more letters. I'll have to write out more of the alphabet.

g h i j k l m n o p q r s t u v w x y z

So *k* is the letter for the last blank.

2 2 2
1 z 3 w 9 t 27 q 81 n __ *k*

Now I'll look at the numbers. *1 3 9 27.* It seems to be repeating patterns of 3. No, it's not repeating patterns of 3; instead, *3, 9,* and *27* are all multiples of 3. Let me see. *3* to *9.* Well, 9 is 3 times 3. *9* to *27.* 27 is 3 times 9. *1* to *3.* 3 is 3 times 1. *27* to *81.* I think 3 times 27 is 81. I'll check. 3 times 7 is 21; and 3 times 2 is 6; carry the 2, is 8. So 3 times 27 is 81. It looks like each number is 3 times the previous number.

To fill the blank I need 3 times 81.

$$\begin{array}{r} 81 \\ \times\ 3 \\ \hline 3 \\ 240 \\ \hline 243 \end{array}$$

$$1\ z\ \overset{2}{3}\ w\ \overset{2}{9}\ t\ \overset{2}{27}\ q\ 81\ n\ 243\ k$$

Pattern description: There are two alternating, independent series, one made up of numbers and the other of letters. Each number is 3 times the previous one. The letters are going backwards, each time skipping 2 letters.

Section 11 Sample Problem

This problem is a little different than the others. However, there is a systematic trend which you can discover through careful analysis, and then use to fill the blanks.

JKLMNO JKLMON JKLOMN JKOLMN ______ ______

Pattern description:

Section 12 Problem Solution

In reading this solution, notice how the problem solver came to identify the trend in small, gradual steps. During her initial reading of the problem, she compared the entries, but only noticed that something was changing near the end of each entry. She didn't even realize that each element involved exactly the same letters. Then she carefully compared the entries again, focusing on the sections that seemed to be changing, and she obtained new information about the types of changes that were occurring. In this way—by making many comparisons and carefully noting the differences from one entry to the next—she was able to pinpoint the trend.

It is generally necessary to make numerous comparisons because your mind can absorb only a limited amount of information at one time. You make comparisons and learn something about the problem. This helps you decide which comparisons to make next. You make more comparisons and you learn more about the problem. Gradually, by noting the similarities, the differences and the changes among the entries, you get a picture of all the relationships existing in the problem. The heart of this process is *numerous, careful comparisons.*

Use this same method in working later problems. Continue to make comparisons until you are sure you understand the relationships completely.

Original Problem

JKLMNO JKLMON JKLOMN JKOLMN _______ _______

Problem Solution

Problem Solver read and thought aloud.

JKLMNO, JKLMON, JKLOMN. So far the first 3 letters have remained the same but the last 3 are changing. *JKOLMN.* Now only the first 2 have remained the same and the others are changing. Now I have to try to find out how they are changing. Looking at the first and second ones, *MNO* turns into *MON.* Then *MON* changes to *OMN.* Now I see that each time there are the same letters, but they are changing position. In going from the first to second, the *O* switched places with *N.* Then in going from the second to the third, the *O* switched places with the *M.*

Let me see. The third is *JKLOMN* and the fourth is *JKOLMN.* Now the *O* switched places with the letter on its left. I guess next time it should switch with the *K.*

> In order to fill the blank, I'll look at the fourth entry. *J, K, O, L, M, N.* Now I have to switch the *K* and the *O*. That gives me *J, O, K, L, M, N.*
>
> *JKLMNO JKLMON JKLOMN JKOLMN JOKLMN* _____
>
> To fill the last blank I'll look at the one I just wrote. It starts *J, O.* I guess I have to switch the *O* with the *J*. That will put the O in front. But I guess that's OK. That's what I have to do to be consistent with the rest of the pattern.
>
> *JKLMNO JKLMON JKLOMN JKOLMN JOKLMN OJKLMN*
>
> Pattern description: All of the entries contain the letters *J, K, L, M, N,* and *O*. In each case, the letters *J, K, L, M,* and *N* remain in alphabetical order. However, the letter *O* switches positions with the letter on its left each time. In other words, the letter *O* moves one position to the left each time.

Section 13 Summary

A good problem solver begins one of these problems by reading along in the series, looking for patterns. He identifies similarities and differences among the entries, and makes mental notes of relationships that he sees. For example, he may observe that the series is composed of letters alternating with numbers, that the numbers seem to be increasing, and that the letters appear to be moving backwards in the alphabet.

As the problem solver gains familiarity with the series, he tries to specify precisely the underlying pattern. He attempts to formulate in his mind some rule which explains how the letters or numbers change in going from one to the next. When he thinks he has found the rule, he checks it against the entire series to be sure it is completely correct. If any part of the series doesn't agree with his rule, he changes his rule until it portrays the entire series accurately.

Finally, when the problem solver is sure his rule is correct, he uses it to fill the blanks, and then writes his rule in the space designated "pattern description."

An important point in working these problems is that the final rule that you formulate and use to fill the blanks must be valid for the entire series. It cannot be only approximately correct. Each of the problems in this chapter was devised according to a definite pattern, and your task is to find a rule which fits (or describes) the pattern.

Quite possibly, as the problems become more difficult, there will be some for which you cannot find the pattern. This does not reflect poor reasoning ability on your part. If you systematically analyze a problem—carefully comparing the elements and looking for relationships—and you still cannot find the overall pattern, it simply means that you have not had background experience with the type of pattern used in that problem. However, if you formulate a sloppy rule—one which does not fit the series perfectly—and if you do not realize that your rule is incorrect because you did not check it carefully—then this is a sure sign of poor reasoning ability. The most serious error that you can make in working these problems is to fill the blanks incorrectly because the rule you formulated does not precisely fit the series. This is pure carelessness. It is much better to leave a question unanswered, than to answer it incorrectly and not recognize your error. The heart of good reasoning is being very careful in formulating your rule, and then thoroughly checking it to determine whether it fits the facts perfectly.

The activities of a good problem solver described above are very similar to the steps preceding many scientific breakthroughs. A scientist such as Charles Darwin observes some facts, gets an idea, collects more facts, formulates the idea precisely into an hypothesis, collects still more facts which either support the hypothesis or lead to a new one, and gradually "discovers" a valid, scientific law. The problems in this chapter can be regarded as miniature problems in scientific analysis and discovery.

Quiz Yourself

1. What is the most serious error that you can make in working the problems in this chapter?

2. Describe in your own words the approach used by good problem solvers in working trend problems.

Homework Assignment

After you have had an opportunity to work some problems in class, make up several new problems of the same type—perhaps one or two a night—for homework. Also write the pattern description for each one on the back of the page. Let your partner try to solve your problems (without seeing the pattern descriptions, of course) and then turn your problems and pattern descriptions in to the instructor so he can give you credit for doing your homework.

PROBLEMS IN IDENTIFYING PATTERNS

While working in pairs, if there is a certain problem that you cannot solve, call your instructor over and he will guide you to the answer.

1. *2 7 10 15 18 23 26 31 34 39* ___ ___ ___

Pattern description:

2. *A B A B B A B A B B A B* ___ ___ ___

Pattern description:

3. *9 a 8 c 7 e 6* ___

Pattern description:

4. *9 12 11 14 13 16 15 18* ___ ___ ___

Pattern description:

5. *B A D C F E H G* ___ ___ ___

Pattern description:

6. *Q Q L Q Q Q Q L L L Q Q L Q Q Q Q L L L Q* ___ ___ ___

Pattern description:

7. *27 24 22 19 17 14 12 9* ___ ___ ___

Pattern description:

8. *A Z B Y C X D* ___ ___ ___

Pattern description:

9. *32 27 29 24 26 21 23* ___ ___ ___

Pattern description:

10. *1 12 121 1212 12121* __ __

Pattern description:

11. *8 10 13 17 22 28 35* __ __ __

Pattern description:

12. *147 144 137 141 138 131 135 132 125* __ __ __

Pattern description:

13. *A Z C X E V G* __ __ __

Pattern description:

14. *J 1 P 3 M 5 J 8 P 1 M 3 J 5 P 8 M 1 J 3* __ __ __

Pattern description:

15. *A1 B2 D4 Z26 J10 C3 A1 G7 N14 M13 E__ A__ B__ __3 __16*

Pattern description:

16. *5 10 20 40 80 160 __ __ __*

Pattern description:

17. *13R 16P 20N 25L 31J ___ ___*

Pattern description:

18. *120 115 109 102 94 ___ ___ ___*

Pattern description:

19. *A B C __ A 1 2 3 2 1 A B C B A 1 __ 3 2 1 __ __ C B A*

Pattern description:

20. *B M N ___ B 2 13 14 13 2 B ___ N M B 2 ___ 14 13 2 ___ ___ N M B*

Pattern description:

21. *2 6 18 54 162 ___ ___*

Pattern description:

22. *A2 3B B4 3A C2 3D D4 3C E2 ___ ___ ___ ___*

Pattern description:

23. *A A B B D D G G K ___ ___ ___*

Pattern description:

24. *A C E C E G E G I G ___ ___ ___*

Pattern description:

25. *64 32 16 8 4* __ __ __ __

Pattern description:

26. *1 11 20* __ *1 A* __ *T K A 1* __ *20 11* __ __ *K T K* __

Pattern description:

27. *ACEG GACE EGAC CEGA* ______ ______

Pattern description:

28. *XOXOOOO OXOXOOO OOXOXOO* ___________ ___________

Pattern description:

29. *2 4 3 6 5 10 9 18 17* __ __ __

Pattern description:

30. *49 48 46 43 39 34* ___ ___ ___

Pattern description:

31. *8 9 7 10 6 11 5 12* ___ ___ ___

Pattern description:

32. *1 3 6 8 16 18 36* ___ ___ ___

Pattern description:

33. *5AA 10BB 12CD 17DG 19EK* ___ ___

Pattern description:

34. *D G F H K J L O N P S R* ___ ___ ___

Pattern description:

35. *1 2 6 24 120 __ 5040 __*

Pattern description:

36. *1 3 4   7 11 18   2 30 32   5 8 13   5 12 __   3 18 __*

Pattern description:

37. *15 4 19   5 80 85   11 2 13   6 3 __   4 __ 14   __ 8 29*

Pattern description:

38. *2 3 5 8 13 21 __ __ __*

Pattern description:

39. *1 2 3 6 11 20 37 __ __ __*

Pattern description:

40. *5000 6000 5900 6900 6800 7800 7700* _____ _____ _____

Pattern description:

41. *13 18 20 19 24 26 25 30 32* __ __ __

Pattern description:

42. *4 14 21 26 36 43 48 58 65* __ __ __

Pattern description:

43. *7 3 4 8 4 5 10 6 7 14 10 11 22* __ __ __

Pattern description:

X. SOLVING MATHEMATICAL WORD PROBLEMS

Section 1 Introduction

People frequently express anxiety and even despair in dealing with mathematical word problems. Generally these feelings stem from unfortunate experiences in early math training. However, just as many people frightened of water have been taught to swim, this chapter has helped many people develop greater confidence and skill in mathematics.

The problems in this chapter are called mathematical word problems. But what does that mean? Simply this: Each problem describes a situation involving numerical relationships. The situation and relationships must first be interpreted and grasped. Then simple arithmetic computations need to be performed to get the answer.

Although these problems are called *mathematical* word problems, they really aren't very different from the nonmathematical problems which you worked earlier. The computations are simple and the use of algebra or formulas is not required. Primarily the problems require that you understand and spell out precisely the situation that is being described. Once a problem has been set up properly, the arithmetic is easy. One of the main things you will learn in this chapter is that if you make a habit of thinking thoroughly and precisely, you can master mathematics. But before turning to the math problems, some general aspects of problem solving will be reviewed.

Section 2 An Expert's Response

As you know, the problem solutions which follow the problems in this book summarize the steps that can be taken in reaching the answers. However, the problem solutions paint a somewhat idealized and perhaps misleading picture. They fail to show the many activities in which good problem solvers become engaged as they analyze a problem and gradually find their way to a solution. Shown on the next page is the actual response of a good problem solver as she solved a problem similar to one you worked earlier. Please read aloud both the comments on the left and the problem solver's response on the right.

Original Problem

> If deleting the letters *R, I,* and *E* from the word *surmise* leaves a meaningful 3-letter word, circle the first *S* in this word surmise. Otherwise circle the *U* in the word *surmise* where it appears for the third time in the exercise.

Problem Solver's Response

Problem Solver began reading aloud.	"If deleting the letters *R, I,* and *E* from the word *surmise* . . .
Problem Solver repeated the letters aloud.	*"R, I, E."*
Problem Solver crossed out the letters R, I, E with her pencil. She then read the remaining letters aloud and pronounced the word they formed.	*"S, U, M, S . . . Sums."*
Problem Solver resumed reading.	"leaves a meaningful 3-letter word, circle . . ."
Problem Solver stopped reading and thought aloud.	I'm getting to the section where I have to do something if the first part is true. I'm a little confused though, maybe I should start reading the sentence from the beginning. No, I'll read the rest of it. "circle the first *S* in this word *surmise.*" I am confused, let me start again. "If deleting the letters *R, I* and *E* from the word *surmise* leaves a meaningful 3-letter word . . ." No, sums is a 4-letter word. "Circle the first *S* in this word *surmise.*" So I won't do this.

"Otherwise circle the *U* in the word *surmise* where it appears for the third time in the exercise.

So this is the direction I'm supposed to follow. Let's see, the word *surmise* appears once . . . twice in the first sentence. Then in the second sentence it appears—3—and that's it. It appears for the third time in the second sentence. So that's the one I should circle. Now I'll reread the last sentence to make sure I followed the directions correctly.

"Otherwise circle the *U* in the word *surmise* where it appears for the third time in the exercise."

I'll scan the sentence again to be sure that the word *surmise* doesn't occur anywhere except the 3 places I found it. Yes, I circled the right one.

Section 3 Another Problem Response

Here is the verbatim response which another problem solver made to a different problem you worked earlier. Again read it aloud, following the problem solver's thoughts and logic.

Original Problem

Sally loaned $7.00 to Betty. But Sally borrowed $15.00 from Estella and $32.00 from Joan. Moreover, Joan owes $3.00 to Estella and $7.00 to Betty. One day the girls got together at Betty's house to straighten out their accounts. Which girl left with $18.00 more than she came with?

Hint: On your diagram, use arrows to show which person has to return money to which other person. Show the direction in which the money must be returned.

Problem Solver's Response

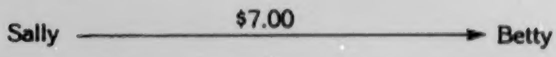
The Problem Solver read aloud.

"Sally loaned $7.00 to Betty. But Sally borrowed $15.00 from Estella and $32.00 from Joan. Moreover, Joan owes $3.00 to Estella and $7.00 to Betty. One day the girls got together at Betty's house to straighten out their accounts. Which girl left with $18.00 more than she came with? Hint: On your diagram, use arrows to show which person has to return money to which other person."

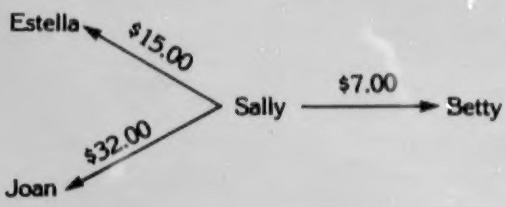
I'll read from the beginning again.

"Sally loaned $7.00 to Betty."

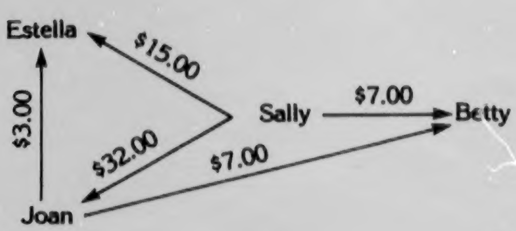
I'll start a diagram with this.


Sally
$7.00
Betty

"But Sally borrowed $15.00 from Estella and $32.00 from Joan." I'll add that to the diagram.


Estella
$15.00
Sally
$7.00
Betty
$32.00
Joan

"Moreover, Joan owes $3.00 to Estella and $7.00 to Betty." I'll add this to the diagram.


Estella
$15.00
$3.00
Sally
$7.00
Betty
$32.00
$7.00
Joan

Problem Solver thought aloud.

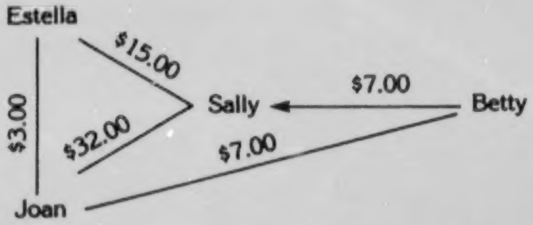
I have all the numbers in, but I don't know who owes whom what. The hint said something about this. I'll read it again.

"Hint: On your diagram, use arrows to show which person has to return money to which other person."

I guess I have to read back over the problem and put in the arrows as I go.

"Sally loaned $7.00 to Betty."

That means that Betty has to return money to Sally. I'll put an arrow pointing to Sally.


Estella
$15.00
$7.00
Sally
Betty
$3.00
$32.00
$7.00
Joan

Problem Solver made the additions to the diagram as he read and thought aloud.

"But Sally borrowed $15.00 from Estella . . ."

The money has to be returned to Estella, so I'll draw the arrow pointing to Estella.

"and $32.00 from Joan."

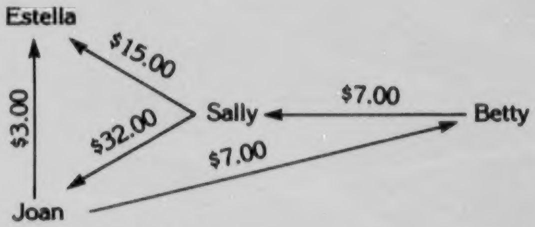
So I'll point the arrow to Joan.

"Moreover, Joan owes $3.00 to Estella . . ."

So I'll point this arrow to Estella.

". . . and $7.00 to Betty."

So this arrow points to Betty.


Estella
$15.00
$7.00
Sally
Betty
$3.00
$32.00
$7.00
Joan

"Which girl left with $18.00 more than she came with?"

Let me figure it out. Looking at my diagram, I see that Joan has to pay $3.00 to Estella and $7.00 to Betty, but she will get $32.00 from Sally. So she has to pay $3.00 plus $7.00 which is $10.00 but she gets $32.00. So she leaves with $22.00.

Betty gets $7.00 from Joan, but pays $7.00 to Sally, so she breaks even.

> Estella gets $3.00 from Joan and $15.00 from Sally. She doesn't pay anything out. $3.00 and $15.00 is $18.00. So she's the one who leaves with $18.00.
>
> Let me check Sally to be sure she isn't also $18.00 ahead. She gets $7.00 from Betty. But she pays $15.00 to Estella and $32.00 to Joan. $32.00 and $15.00 is $47.00. She gets $7.00. So she leaves with $40.00 less than when she came.

Section 4 Concern For Accuracy, Step-by-Step Analysis And Subvocal Speech

Two characteristics of good problem solvers were very evident in the responses you just read.

First is carefulness: the concern and quick retracking when ideas became confusing; the rechecking, reviewing and rereading to be sure that errors hadn't crept in, that nothing had been overlooked.

Second is the step-by-step approach. An important example of this was when the problem solvers restated ideas in their own words, in a form which was clearer or more useful to them. For instance, at one place in the last section the problem solver read "Sally loaned $7.00 to Betty." He changed this to "That means Betty has to return money to Sally." This shows that the problem solver went through two steps in representing the information in his diagram. First he translated the original statement to one which was closer to the form he needed for the diagram. Then he incorporated the new statement into his diagram. With the new statement he could more easily see that on the diagram he had to draw the arrow pointing toward Sally. Restating ideas is an important way in which good problem solvers use the step-by-step method to analyze the fine details of a problem.

While on this topic, it should be emphasized that restating ideas—and "talking to themselves" while thinking—is not something good problem solvers do only when they are asked to work a problem aloud. Studies using electronic amplifying equipment (to monitor speech muscle activity) reveal that good problem solvers talk to themselves while they solve problems. They repeat information, rephrase it, weigh it, compare different facts, express thoughts like "I better read the first sentence again," and in general, clarify ideas for themselves. This talking, which is not done aloud, is called covert or subvocal speech.

Quiz Yourself

What is subvocal speech? What do good problem solvers talk to themselves about while solving problems?

Section 5 Sample Ratio Problem

Here is a simple ratio problem. If you are using this book in a class, one student should work the problem aloud.

> A train travels 5 mi in 3 min. At this same speed, how far will it travel in 6 min?

This problem tells you the distance that the train travels in 3 minutes—and then asks you to compute the distance it travels in 6 minutes. Six minutes is twice as much time as 3 minutes. If the train travels for twice as much time, it will go twice as far.

This problem is called a *ratio* problem because you first compute the ratio of 6 minutes to 3 minutes. Once you see that 6 minutes is twice as long as 3 minutes, you use that information to calculate the answer.

Ratio problems can become more complicated than this one, and generally there is more than one way to solve them. The ability to solve ratio problems is important in many high school and college courses.

Section 6

Here is another ratio problem. In a class, one student should work it aloud at the chalkboard.

> A train can travel 10 mi in 4 min. How far will it travel in 14 min?

Section 7 Several Approaches

Here is the problem you were asked to solve in section 6:

> A train can travel 10 mi in 4 min. How far will it travel in 14 min?

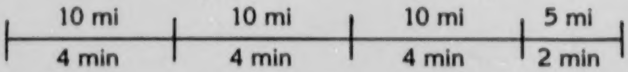
There is more than one way to solve this problem. We will look at three ways, which we will call solutions 1, 2, and 3. Each solution can be viewed in terms of the underlying logic, and in terms of the mathematical computations. Looking at the solution in terms of the underlying logic is slower and less elegant. But in a sense it is more important. People who try to apply mathematical formulas without comprehending the underlying logic of a problem stand a good chance of using an incorrect formula and arriving at a wrong answer. Therefore we will begin with the logic and then look at the mathematical computations.

Solution 1

Logic of the Solution: Here is one way to look at the problem. If the train can travel 10 miles in 4 minutes, then it can travel 20 miles in 8 minutes, and 30 miles in 12 minutes. If it travels still another 2 minutes (to make a total of 14 minutes) it will go 5 more miles, for a total of 35 miles. This is shown below:

4 min	10 mi
4 min	10 mi
4 min	10 mi
2 min	5 mi
14 min	35 mi

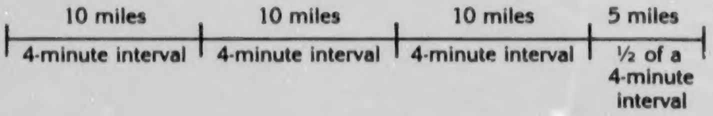
This is also represented in the following diagram:



Mathematical Solution of the Problem: Notice that in constructing the table above we counted off how many times 4 minutes goes into 14 minutes. We found that it does three times, with 2 minutes left over. At the same time we counted off the same number of 10-mile sections, and concluded that the train travels 35 miles in 14 minutes. We do the same thing arithmetically when we divide 4 minutes into 14 minutes and then multiply this by 10 miles. Thus:

$$14/4 = 3\tfrac{1}{2} \qquad 3\tfrac{1}{2} \times 10 = 35$$

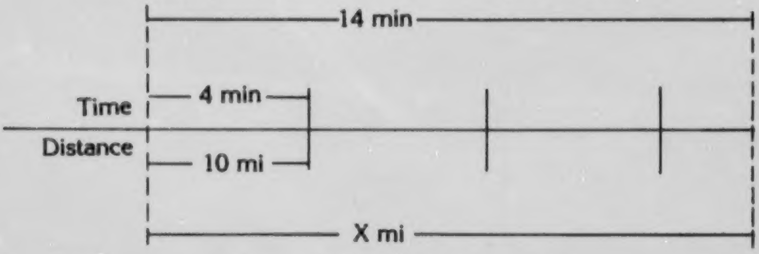
The important thing to understand is that this procedure of dividing and then multiplying is really a shortcut method of spelling out the entire situation, as was done in the table and the diagram above. The diagram shows that we are thinking in terms of 4-minute intervals. In other words, 12 minutes is exactly three 4-minute intervals; and 2 minutes is one-half of a 4-minute interval. Moreover, the train travels 10 miles in each 4-minute interval, and 5 miles in one-half of a 4-minute interval. So it goes 35 miles total.



Solution 2

Logic of the Solution: Another way to solve the problem is to think in terms of ratios that are equal to each other. This approach requires more mathematical background and experience than the other two solutions. Don't use this method with any of the later problems in this chapter unless you are absolutely certain you understand exactly what you are doing.

The logic of the solution is shown in the following diagram:



Note the *X* in the diagram. This is what the problem asks you to find, the distance traveled by the train in 14 minutes.

From the diagram you can see that since 14 minutes is 3½ times more than 4 minutes, the unknown distance *X* must be 3½ times larger than 10 miles. In other words, the ratio of 14 minutes to 4 minutes is equal to the ratio of *X* miles to 10 miles. With this idea in mind we can write the following equation:

$$\frac{14}{4} = \frac{X}{10}$$

On the left side of the equation we have the ratio of 14 minutes to 4 minutes. And on the right side we have the ratio of *X* miles to 10 miles. The equal sign means that these two ratios must be numerically equal.

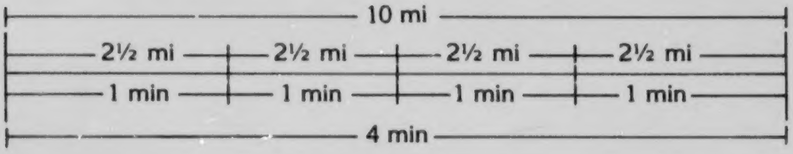
Mathematical Solution: Once you have set up this equation of ratios, you find *X* by using simple arithmetic and algebra. Here are steps you could employ:

1. Initial equation: $$\frac{14}{4} = \frac{X}{10}$$
2. Multiply both sides of the equation by 10 so that just *X* will remain on the right side. $$\frac{10(14)}{4} = \frac{X\,(10)}{10}$$
3. Cancel on both sides of the equation. $$\frac{\overset{(5)}{\cancel{10}}\,\overset{(7)}{(\cancel{14})}}{\underset{1}{\cancel{4}}} = \frac{X\,\overset{1}{(\cancel{10})}}{\underset{1}{\cancel{10}}}$$

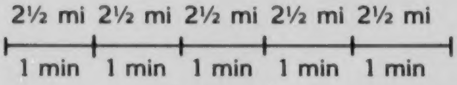
4. Multiply the numbers to get *X*. ([illegible]7) = 35 = X

Solution 3

Logic of the Solution: A third approach begins by asking how many miles the train travels in 1 minute, and then multiplying this by 14 minutes to find the total distance traveled by the train. The following diagram shows that since the train travels 10 miles in 4 minutes, it must travel 2½ miles in 1 minute.



Once we know that the train travels 2½ miles in 1 minute, it is easy to find out how far it travels in 14 minutes. We just add up 14 of these intervals, as shown below.



etc. for a total of 14 minutes.

Mathematical Solution: First you need to determine how many miles the train travels each minute:

$$\frac{10 \text{ mi}}{4 \text{ min}} = 2\tfrac{1}{2} \text{ mi per min}$$

Once you know that the train travels 2½ miles each minute, you can multiply this by 14 to find how far it travels in 14 minutes.

$$2\tfrac{1}{2} \text{ mi each min} \times 14 \text{ min} = 35 \text{ mi}$$

Summary

All three solutions are correct. They use different computational approaches and formulas, but they are based on basically the same picture of what happens as the train's traveling time increases from 4 minutes to 14 minutes.

Section 8 Sample Problem

Try this problem. It is in some ways similar but in other ways different from the last one. In a class, one student should solve the problem aloud at the chalkboard, explaining his logic completely.

> A certain ruler, which is supposed to be 12 in long, is warped and is actually just 11½ in. If you measure off 4 ft of string with this ruler, how long would the string really be?

Section 9 Problem Solution

Original Problem

> A certain ruler, which is supposed to be 12 in long, is warped and is actually just 11½ in. If you measure off 4 ft of string with this ruler, how long would the string really be?

Problem Solution

A good way to begin this problem is to imagine yourself actually taking a ruler and measuring off 4 feet of string. You might picture a ball of string on a table and see yourself pulling out the string and laying off 1 foot at a time using the ruler.

The ruler is supposedly 12 inches long, and 12 inches is 1 foot. Each time you measure off a section of string equal to the length of the ruler, you supposedly have 1 foot of string. Since you want 4 feet, you will have to do this four times.

However, the ruler is warped, so each section of string is only 11½ inches long. Therefore when you measure off four sections of string, the total length is:

$$4 \times 11\tfrac{1}{2} \text{ in} = 46 \text{ in}$$

There are several alternate ways of doing this problem. In picturing yourself laying off four sections of string with the ruler, you might calculate how much your accumulated error is.

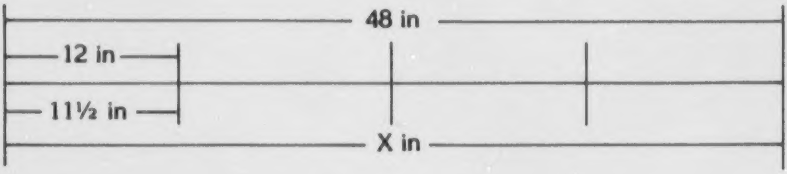
The ruler is 11½ inches, so each time you lay off a section of string with the ruler you are ½ inch short of the full 12 inches. When you lay off four sections of string with the ruler, you will be four times that short:

$$4 \times \tfrac{1}{2} \text{ in short} = 2 \text{ in short}$$

Since you are 2 inches short of the full 48 inches, your string will actually be 46 inches long.

$$48 \text{ inches} - 2 \text{ inches} = 46 \text{ inches}$$

A third way of approaching this problem is to look at it in terms of ratios. In the diagram below we let *X* inches be the unknown length of the string measured off with the warped ruler. The diagram shows that there are as many 12-inch sections in 48 inches as there are 11½ inch sections in *X* inches.



From the diagram we see that because 48 divided by 12 is four, we know that *X* divided by 11½ must also be four. The ratio of 48 to 12 is equal to the ratio of X to 11½. In mathematical symbols we write that this way:

$$\frac{48}{12} = \frac{X}{11\frac{1}{2}}$$

To solve this equation you could employ the following steps:

1. Initial equation $$\frac{48}{12} = \frac{X}{11\frac{1}{2}}$$
2. Multiply both sides of the equation by 11½, to obtain just *X* on the right side. $$\frac{(11\frac{1}{2})48}{12} = \frac{X\,(11\frac{1}{2})}{11\frac{1}{2}}$$
3. Cancel. $$\frac{(11\frac{1}{2})\overset{4}{\cancel{48}}}{\underset{1}{\cancel{12}}} = \frac{X\,(\overset{1}{\cancel{11\frac{1}{2}}})}{\underset{1}{\cancel{11\frac{1}{2}}}}$$
4. Multiply to get *X*. $$(11\tfrac{1}{2})4 = 46 = X$$

Section 10 Sample Problem

Some of the math word problems in this chapter are quite different from the ratio or rate problems you have just worked. Here is an example. In a class, one student should solve the problem aloud at the chalkboard.

> Ted's weekly income is $100.00 less than double Gary's weekly income. If Ted makes $500.00 a week, what does Gary make?

Section 11 Problem Solution

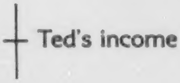
Original Problem

> Ted's weekly income is $100.00 less than double Gary's weekly income. If Ted makes $500.00 a week, what does Gary make?

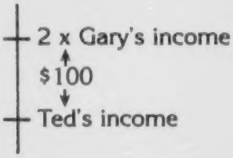
Problem Solution

Whenever it is possible to represent the ideas of a math problem with a diagram, it is generally useful to do so. We will use a diagram to help keep track of the facts in this problem.

The first sentence says Ted's weekly income is $100.00 less than double Gary's weekly income. We'll draw a line to represent income and then place Ted on it.

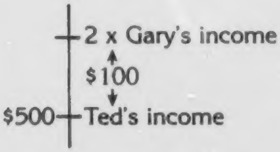


Since Ted's income is $100.00 less than double Gary's, it means that on the diagram Ted's income should be below (less) double Gary's income. Therefore we'll place "double Gary's income" above Ted's income, separated by $100.00.

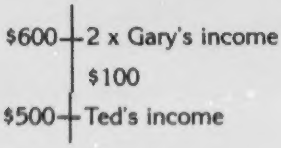


Check the diagram. Does it show what it is supposed to show, namely that Ted makes $100.00 less than double Gary?

Finally, the problem says that Ted makes $500.00. We'll add that to the diagram.



From the diagram we see that double Gary's income is $100.00 more than Ted's income—which means it is $600.00.



Since double Gary's income is $600.00, his actual income must be one-half of that:

½($600) = $300

In working this problem there is a common error made, especially if a diagram is not used to help keep track of the relationships. As soon as beginners see the words "less than" they often automatically conclude that they should subtract. Their reasoning goes something like this: "Ted's income is $100.00 less than double Gary's income. Ted makes $500.00 a week; $100.00 less than that is $400.00. So double Gary's income is $400.00 and ½ of that is $200.00."

Do you see where the error was made in this train of thought? Before continuing, review and explain it in your own words.

This faulty reasoning comes from dealing carelessly or superficially with the words "less than," rather than spelling out exactly which income is less than which. In all of the problems that follow, spell out fully and precisely the relationships among the facts. And whenever you feel a little doubtful or confused, try to use a diagram.

Section 12 Sample Problem

Work this problem by representing the facts of the first sentence in one diagram, and then the second sentence in a separate diagram. In a class, one student should work the problem aloud at the chalkboard.

> Paul makes $25.00 a week less than the sum of what Fred and Carl together make. Carl's weekly income would be triple Steven's if he made $50.00 more a week. Paul makes $285.00 a week and Steven makes $75.00 a week. How much does Fred make?

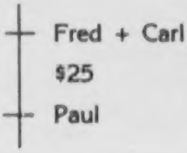
Section 13 Problem Solution

Original Problem

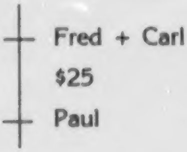
> Paul makes $25.00 a week less than the sum of what Fred and Carl together make. Carl's weekly income would be triple Steven's if he made $50.00 more a week. Paul makes $285.00 a week and Steven makes $75.00 a week. How much does Fred make?

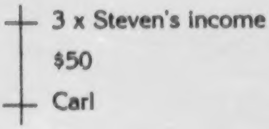
Problem Solution

The first sentence says Paul makes $25.00 a week less than the incomes of Fred and Carl combined. So we'll make a diagram with Paul placed $25.00 below Fred plus Carl.

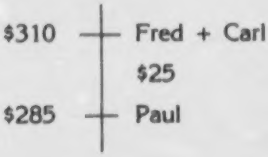

Fred + Carl
$25
Paul

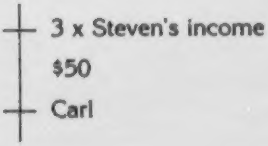
The second sentence says Carl's weekly income would be triple Steven's if he made $50.00 more a week. In other words, if we go up $50.00 from Carl's income it will bring us to triple Steven's income. This is shown in the diagram on the right.


Fred + Carl
$25
Paul

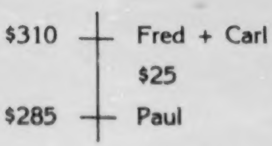

3 x Steven's income
$50
Carl

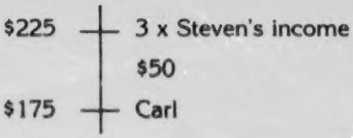
The problem says Paul makes $285.00 a week. When we add that to the diagram, we see that Fred plus Carl combined make $310.00.


$310
Fred + Carl
$25
$285
Paul


3 x Steven's income
$50
Carl

The problem also says that Steven makes $75.00 a week. Therefore triple Steven's income is: 3 x $75 = $225. The diagram on the right shows that Carl makes $50.00 less than this, so Carl must make $175.00 a week.





We now know that Carl's income is $175.00 a week, and we also know that the sum of Fred's plus Carl's incomes is $310.00 a week. From this we can determine Fred's income by subtraction.

$$\$310 - \$175 = \$135$$

Section 14 Procedure

This section reviews briefly the procedure for solving math word problems. As you read the passage, try to think of situations in which you could apply these activities.

1. Vocalize (think-aloud) all of your thoughts, decisions, analyses and conclusions. Vocalize how you are starting the problem, questions you are asking yourself, steps you are taking in breaking the problem into parts, conclusions you are drawing—everything. If you have to add some numbers, add them aloud. If you perform any other mental operations (such as translating an unfamiliar word to a familiar word, or visualizing a picture of a relationship described in the text), perform these operations aloud. No matter how confusing you find a problem, don't work any of it silently. Start talking right from the beginning and keep talking.

2. Try to adopt the step-by-step analytical procedure and the various other techniques that good problem solvers use. Break a problem into parts. Work one part accurately and then move on to the next part. Translate unfamiliar or unclear phrases into your own words. Interpret phrases and sentences in the light of the entire problem. Visualize relationships presented in the text.

3. Be extremely accurate. Continually check your thinking. In the back of your mind there should always be the thought: "Is that entirely correct? . . . Is that completely accurate?" Never work so quickly that it leads to errors. Work everything slowly and carefully. Give sufficient time to all parts of the problem. Never just give up on a problem and guess an answer. Always try to reason the problem out.

4. While another student is working a problem, check his accuracy so that he will learn to think with more precision and thoroughness. In addition, in

your own mind contrast his method with the way you might attack the same problem. How might you break the problem down more completely into sub-problems? What other steps might you take? How might you clarify your understanding of some portion of the problem by referring to other sections of the problem? How might you visualize certain relationships? Would you work more carefully and accurately? In other words, try to imagine ways in which you might attack the problem more effectively.

Homework Assignment

After you have had an opportunity to work some problems in class, make up several problems of the same type for homework. Also, write out the steps that you would use to solve the problems.

MATH WORD PROBLEMS

Introduction

Your instructor may ask you to work in pairs, taking turns solving the following problems aloud. In working the problems, make sure you understand the underlying logic before you apply any formulas. First take some time to spell out in your mind (or in a table or diagram) what the actual situation is, and only then start to make your mathematical computations. This is the key to working mathematical word problems correctly.

Special Instruction for Listeners

If your partner performs computations or applies formulas which are inappropriate and lead to wrong answers—because he has not spelled out situations with full understanding—stop him and *insist* that he show you a table or diagram which illustrates, step-by-step, the relationships between the facts in the problem. Stopping your partner and requesting a full explanation of why he is performing certain computations is your responsibility in helping him become a good mathematical problem solver.

Problem 1

John can run 7 ft in the time that Fred runs 5 ft. How far will John run in the time that Fred runs 15 ft?

Original Problem

John can run 7 ft in the time that Fred runs 5 ft. How far will John run in the time that Fred runs 15 ft?

Problem Solution

Logic of the Solution: If John can run 7 feet when Fred runs 5, then John can run 14 feet when Fred runs 10, and he can run 21 feet when John runs 15. This is shown in the table below.

Fred	John
5	7
5	7
5	7
15	21

Mathematical Solution:

Step 1. When Fred runs 15 feet, how many times further does he run compared to when he runs only 5 feet? To answer this you divide:

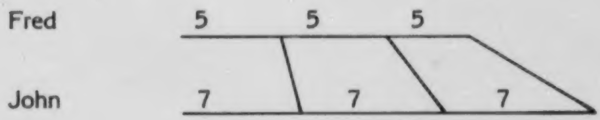
$$15/5 = 3.$$

Step 2. If Fred runs three times as far, he must run three times as long.

Step 3. When John also runs three times as long, how much distance does he cover? The answer is 7 x 3 = 21 feet.

This diagram shows that every time Fred runs 5 ft, John runs 7 ft.

When Fred runs 15 ft, John runs 21 ft.



Problem 2

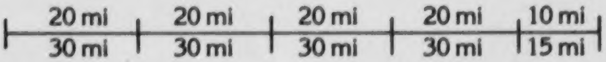
A train travels 30 mi in the time a car travels 20 mi. At that rate, how far will the train travel when the car travels 90 mi?

Original Problem

A train travels 30 mi in the time a car travels 20 mi. At that rate, how far will the train travel when the car travels 90 mi?

Problem Solution

Logic of the Solution: If the train travels 30 miles when the car travels 20, then the train travels 60 miles when the car travels 40, it travels 90 miles when the car travels 60, it travels 120 miles when the car travels 80, and it travels 135 miles when the car travels 90. This is shown below.



Mathematical Solution:

Step 1. We need to know how many times as far the car travels when it goes 90 miles compared to when it goes 20 miles.

$$90/20 = 4\tfrac{1}{2} \text{ or } 4.5$$

Step 2. If the car travels 4.5 times as far in going 90 miles, it must travel 4.5 times as long. When the train travels 4.5 times as long, it goes this far: 4.5 x 30 miles.

$$\begin{array}{r} 30 \\ \times \quad 4.5 \\ \hline 150 \\ 120 \\ \hline 135.0 \end{array}$$

The train travels 135 miles when the car travels 90 miles.

Problem 3

Be careful to notice the difference between feet and yards in this problem.

If Fred runs 4 yds in the time John runs 9 ft, how many ft does John run when Fred runs 120 ft?

Original Problem

If Fred runs 4 yds in the time John runs 9 ft, how many ft does John run when Fred runs 120 ft?

Problem Solution

Step 1. One yard is 3 feet. So 4 yards equals 12 feet.

The problem can be restated like this:

If Fred runs 12 ft in the time John runs 9 ft, how many ft does John run when Fred runs 120 ft?

Step 2. We need to know how many times as far Fred goes when he runs 120 feet compared to when he runs 12 feet.

$$\frac{120}{12} = 10$$

Step 3. If Fred travels 10 times as far in going 120 feet, he must travel 10 times as long. During that same time John would also run 10 times as far:

$$10 \times 9 \text{ feet} = 90 \text{ feet}$$

Problem 4

A man runs 1 mi in 10 min and a car goes 50 mi an hr. At these rates, how far does the man go when the car goes 150 mi?

Original Problem

A man runs 1 mi in 10 min and a car goes 50 mi an hr. At these rates, how far does the man go when the car goes 150 mi?

Problem Solution

Step 1. One way to work this problem is to determine how much time the car takes to go 150 miles, and then find how far the man runs in the same time.

Step 2. The car goes 150 miles, and it covers 50 miles each hour. Therefore the number of hours it travels is:

$$\frac{150}{50} = 3 \text{ hrs}$$

This means the man also runs for 3 hours.

Step 3. The problem says the man runs 1 mile every 10 minutes. Since an hour has 60 minutes (which is 6 times as long as 10 minutes) he runs 6 miles an hour. Therefore in 3 hours he runs:

$$3 \text{ hrs} \times 6 \text{ mi per hr} = 18 \text{ mi}$$

Problem 5

A car travels 40 mi an hr and a plane travels 10 mi a min. How far will the car travel while the plane travels 450 mi?

Original Problem

A car travels 40 mi an hr and a plane travels 10 mi a min. How far will the car travel while the plane travels 450 mi?

Problem Solution

Step 1. We can work this problem by determining how much time the plane takes to go 450 miles, and then finding out how far the car goes in the same time.

Step 2. Since the plane goes 450 miles, and it travels 10 miles each minute, it's traveling time is:

$$\frac{450}{10} = 45 \text{ min}$$

Step 3. This means the car also travels for 45 minutes. The problem says it travels 40 miles an hour. Let's figure out how far it travels per minute. Since there are 60 minutes in an hour, the car will travel 1/60 of 40 miles in a minute.

$$\frac{1}{60}(40 \text{ mi}) = \frac{2}{3} \text{ mi}$$

Step 4. The car travels 2/3 of a mile each minute. In 45 minutes it will go:

$$45 \text{ min} \times \frac{2}{3} \text{ mi per min} = 30 \text{ mi}$$

Problem 6

Clock *A* keeps perfect time whereas clock *B* runs fast. When clock *A* says 6 min have passed, clock *B* says 8 min have passed. How many minutes have really passed when clock *B* says 56 min have passed?

Original Problem

> Clock *A* keeps perfect time whereas clock *B* runs fast. When clock *A* says 6 min have passed, clock *B* says 8 min have passed. How many minutes have really passed when clock *B* says 56 min have passed?

Problem Solution

Step 1. The question is how many minutes have passed according to clock *A* (which is accurate) when clock *B* shows that 56 minutes have passed.

Step 2. When clock *B* says 8 minutes have passed, actually only 6 minutes have passed. When clock *B* says 16 minutes have passed, only 12 minutes have passed.

Clock B	*Clock A (Accurate)*
8	6
8	6
—	—
—	—

Step 3. It is necessary to find out how many 8-minute intervals there are in 56 minutes.

$$\frac{56}{8} = 7$$

Step 4. So the actual time that passed is 7 intervals of 6 minutes each.

$$7 \times 6 \text{ minutes} = 42 \text{ minutes}$$

Step 5. The actual time that passed is 42 minutes.

Clock B	*Clock A (Accurate)*
8	6
8	6
8	6
8	6
8	6
8	6
8	6
56	42

Problem 7

Clock *A* keeps perfect time whereas clock *B* runs fast. When clock *A* says 4 min have passed, clock *B* says 6 min have passed. How many minutes have really passed when clock *B* says 27 min have passed?

Original Problem

Clock *A* keeps perfect time whereas clock *B* runs fast. When clock *A* says 4 min have passed, clock *B* says 6 min have passed. How many minutes have really passed when clock *B* says 27 min have passed?

Problem Solution

Step 1. The question is how many minutes have passed according to clock *A* (which is accurate) when clock *B* shows that 27 minutes have passed.

Step 2. When clock *B* says 6 minutes have passed, actually only 4 minutes have passed. When clock *B* says 12 minutes have passed, only 8 minutes have passed.

Clock B	*Clock A (Accurate)*
6	4
6	4
—	—
—	—

Step 3. It is necessary to find out how many 6-minute intervals there are in 27 minutes.

$$\frac{27}{6} = 4\frac{1}{2}$$

Step 4. So the actual time that passed is 4½ intervals of 4 minutes each.

4½ x 4 minutes = 18 min

Step 5. The actual time that passed is 18 minutes.

Clock B	*Clock A (Accurate)*
6	4
6	4
6	4
6	4
3	2
27	18

Problem 8

A certain clock runs fast, gaining 6 min every hour. If it is set correctly at 3:00, what time will it show when the correct time is 7:30?

Original Problem

A certain clock runs fast, gaining 6 min every hour. If it is set correctly at 3:00, what time will it show when the correct time is 7:30?

Problem Solution

Step 1. From 3:00 to 7:30 is 4½ hours.

Step 2. The clock gains 6 minutes every hour. In 4 hours it will gain 24 minutes, and in ½ hour it will gain 3 minutes. So in 4½ hours it will gain 27 minutes.

$$4\frac{1}{2} \times 6 = 27$$

Step 3. 7:30 plus 27 minutes is 7:57.

The clock will read 7:57.

True Time	*Clock Reads*
4:00	4:06
5:00	5:12
6:00	6:18
7:00	7:24
7:30	7:57

Problem 9

A certain clock runs fast; it indicates 1 hr has passed when actually only 56 min have passed. If it is set correctly at 1:00, what is the correct time when it reads 6:30?

Original Problem

A certain clock runs fast; it indicates 1 hr has passed when actually only 56 min have passed. If it is set correctly at 1:00, what is the correct time when it reads 6:30?

Problem Solution

Step 1. Every time the clock shows an hour having passed, actually 4 minutes less than an hour has passed. When the clock shows 2 o'clock, it is really 1:56. When it shows 3:00, it is really 2:52.

Clock	*Correct Time*
1:00	1:00
2:00	1:56
3:00	2:52

Step 2. When the clock moves from 1:00 to 6:30, it moves 5½ hours. For each hour on the clock, the true time that passed is 4 minutes less than an hour. So when the clock says that 5½ hours have passed, the true time that passed is less than that by the amount:

5½ hrs x 4 min less per hour = 22 min less

Step 3. When the clock reads 6:30, the true time is 22 minutes less than this:

6:30 minus 22 min = 6:08

Clock	*Error*	*Correct Time*
1:00		1:00
2:00	4 min.	1:56
3:00	8 min.	2:52
4:00	12 min.	3:48
5:00	16 min.	4:44
6:00	20 min.	5:40
6:30	22 min.	6:08

Problem 10

Clock *A* loses 4 min every half hour, and clock *B* gains 5 min every 2 hrs. Both clocks are set correctly at 5:00 P.M. How many minutes apart are they when the true time is 9:00 P.M.?

Original Problem

Clock *A* loses 4 min every half hour, and clock *B* gains 5 min every 2 hrs. Both clocks are set correctly at 5:00 P.M. How many minutes apart are they when the true time is 9:00 P.M.?

Problem Solution

Step 1. From 5:00 P.M. to 9:00 P.M. is a total period of 4 hours.

Step 2. Clock *A* loses 4 minutes every half hour. That means it loses 8 minutes every hour. So in 4 hours it will be behind by this amount:

4 hrs x 8 min every hour = 32 min

Step 3. Clock *B* gains 5 minutes every 2 hours. Therefore in 4 hours it gains 10 minutes.

Step 4. So far we have determined that clock *A* will be 32 minutes behind when the correct time is 9:00 P.M., and clock *B* will be 10 minutes ahead. That means they will be 42 minutes apart.

Problem 11

A 12-in ruler is poorly constructed and is really 12½ in long. You measure off what you believe is 5 yrds of string with this ruler. What is the true length of the string?

Original Problem.

A 12-in ruler is poorly constructed and is really 12½ in long. You measure off what you believe is 5 yrds of string with this ruler. What is the true length of the string?

Problem Solution

Step 1. Each time you measure off a section of string equal in length to the ruler, it is really 12½ inches long.

Step 2. You want 5 yards of string. Since there are 3 feet in a yard, this is 15 feet of string.

Step 3. When you measure off 15 sections of string, each equal to the length of the ruler, the total length of string is:

$$15 \times 12\tfrac{1}{2} \text{ in} = 187\tfrac{1}{2} \text{ in}$$

$$\begin{array}{r} 15 \\ \times\ 12.5 \\ \hline 75 \\ 30 \\ 15 \\ \hline 187.5 \end{array}$$

Problem 12

How many inches short would a piece of cloth be if you measure off 24 ft of cloth with a yardstick that is warped and is 2 in short?

Note: A yardstick is 1 yrd, and 1 yrd equals 3 ft.

Original Problem

How many inches short would a piece of cloth be if you measure off 24 ft of cloth with a yardstick that is warped and is 2 in short?

Note: A yardstick is 1 yrd, and 1 yrd equals 3 ft.

Problem Solution

Step 1. Each time you measure off a yardstick's length of material it is 2 inches short. When you measure off 2 yards it is 4 inches short, when you measure off 3 yards it is 6 inches short.

Measured Length (yrd)	*Error (in)*
1	2
2	4
3	6

Step 2. It is necessary to determine how many times the yardstick would be used in measuring off 24 feet of material. One yard is 3 feet. Two yards is 6 feet. Three yards is 9 feet. Four yards is 12 feet. Five yards is 15 feet. Six yards is 18 feet. Seven yards is 21 feet. Eight yards is 24 feet.

So the yardstick will be applied eight times, and each time it is in error by 2 inches. Therefore, the total error will be:

$$8 \times 2 \text{ in} = 16 \text{ in}$$

Step 3. Thus, the piece of material would be 16 inches short.

Measured Length (yrd)	*Error (in)*
1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16

Problem 13

Bob owns 6 suits, 3 less than Harvey and twice as many as Phil. Gene owns 3 times as many suits as Harvey. How many suits each do Gene and Phil own?

Note: Knowledge of algebra is not required in solving this problem or the ones that follow. All that is required is that you read the problem carefully, determine what it is asking, determine what information and what relationships are given, and then work through the relationships accurately.

Original Problem

Bob owns 6 suits, 3 less than Harvey and twice as many as Phil. Gene owns 3 times as many suits as Harvey. How many suits each do Gene and Phil own?

Problem Solution

Step 1. Bob owns 6 suits.

Bob—6

Step 2. Bob owns 3 suits less than Harvey. That means Harvey owns 3 more suits than Bob, so Harvey owns 9 suits.

Bob—6 Harvey—9

Step 3. Bob owns twice as many suits as Phil. This means Phil owns only half as many suits as Bob. Since Bob owns 6 suits, Phil owns 3 suits.

Bob—6 Harvey—9 Phil—3

Step 4. Gene owns 3 times as many suits as Harvey. Harvey owns 9 suits, so Gene owns 27 suits.

Bob—6 Harvey—9 Phil—3 Gene—27

Step 5. How many suits each do Gene and Phil own?

Gene owns 27, Phil owns 3.

Problem 14

The number of cows owned by farmer Smith is the number owned by farmer Thompson divided by the number owned by farmer Jones. Farmer Thompson, who owns 42 cows, would own 8 times as many cows as farmer Jones if he owned 14 more cows. How many cows does farmer Smith own?

Original Problem

> The number of cows owned by farmer Smith is the number owned by farmer Thompson divided by the number owned by farmer Jones. Farmer Thompson, who owns 42 cows, would own 8 times as many cows as farmer Jones if he owned 14 more cows. How many cows does farmer Smith own?

Problem Solution

Step 1. The problem asks how many cows farmer Smith owns.

Step 2. The number of cows owned by Smith is the number owned by Thompson divided by the number owned by Jones.

$$\text{Smith's cows} = \frac{\text{Thompson's cows}}{\text{Jones' cows}}$$

Step 3. Thompson has 42 cows.

Thompson—42 cows

Step 4. If Thompson owned 14 more cows he would own eight times as many cows as Jones.

$$\text{Thompson's cows} + 14 = 8 \times \text{Jones' cows}$$

Step 5. If Thompson owned 14 more cows he would own:

$$42 + 14 = 56 \text{ cows}$$

Step 6. So 56 cows is eight times as many cows as Jones owns.

$$56 = 8 \times \text{Jones' cows}$$

Step 7. To find Jones' cows divide 56 by 8.

$$\text{Jones's cows} = \frac{56 \text{ cows}}{8} = 7 \text{ cows}$$

Step 8. To get the number of cows owned by Smith, the division shown in step 2 must be carried out.

$$\text{Smith's cows} = \frac{42}{7} = 6 \text{ cows}$$

Step 9. Smith owns 6 cows.

Problem 15

If Bob's weekly income doubled he would be making $50.00 a week more than Tom. Bob's weekly income is $70.00 more than one-half of Phil's. Phil makes $180.00 a week. How much does Tom make?

Original Problem

If Bob's weekly income doubled he would be making \$50.00 a week more than Tom. Bob's weekly income is \$70.00 more than one-half of Phil's. Phil makes \$180.00 a week. How much does Tom make?

Problem Solution

Step 1. Strategy for working this problem: The problem asks for Tom's income. It gives Phil's income as \$180.00. It also tells how to get from Phil's income to Bob's income, and then from Bob's income to Tom's income.

Step 2. It says Phil's income is \$180.00 and it says Bob's income is \$70.00 more than one-half of this. So if Phil's income is divided in half, and then \$70.00 is added, we will have Bob's income.

$$\frac{\text{Phil's income}}{2} + \$70 = \text{Bob's income}$$

$$\frac{\$180}{2} + \$70 = \$90 + \$70 = \$160 = \text{Bob's income}$$

Step 3. Thus, Bob's income is \$160.00 a week.

Step 4. It says that double Bob's income is \$50.00 more than Tom's.

2 x Bob's income is \$50.00 more than Tom's income.

Step 5. Double Bob's income is: 2 x \$160 = \$320.

Step 6. So, \$320.00 is \$50.00 more than Tom's income. Therefore, Tom's income is \$320 − \$50 = \$270.

Problem 16

Paul owns 4 more than one-half as many books as Pete. Paul owns 32 books. How many books does Pete own?

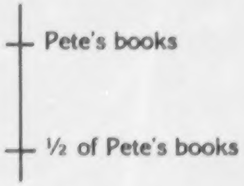
Note: After you solve the problem, please read the problem solution—even if your answer is correct. The problem solution illustrates a principle which is used in the remaining four problems of the program.

Original Problem

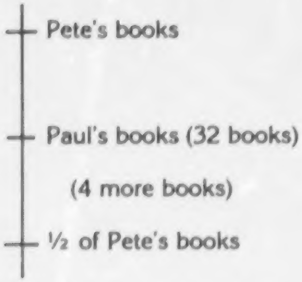
Paul owns 4 more than one-half as many books as Pete. Paul owns 32 books. How many books does Pete own?

Problem Solution

Step 1. A diagram can be used to clarify the relationship between Paul's books and Pete's books. Starting at Pete's books, we take one-half of them.


Pete's books
½ of Pete's books

If we now add 4 books, we have Paul's books (which the problem says is 32 books).


Pete's books
Paul's books (32 books)
(4 more books)
½ of Pete's books

Step 2. To solve the problem work backwards. The diagram shows that if 4 books are taken from Paul he will have exactly one-half as many as Pete.

$$\text{Paul's books} - 4 = \tfrac{1}{2} \text{ of Pete's books}$$

$$32 - 4 = 28 = \tfrac{1}{2} \text{ of Pete's books}$$

Step 3. One-half the number of books owned by Pete is 28. So Pete owns twice this number.

$$\text{Pete's books} = 2 \times 28 = 56$$

Step 4. Pete owns 56 books.

Problem 17

Joan owns 1 more than 3 times as many dresses as Sue. Joan owns 28 dresses. How many dresses does Sue own?

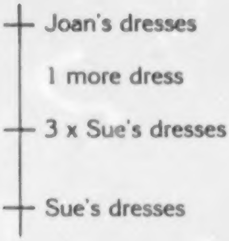
Note: Make a diagram in the space below if it helps your thinking.

Original Problem

Joan owns 1 more than 3 times as many dresses as Sue. Joan owns 28 dresses. How many dresses does Sue own?

Problem Solution

Step 1. The relationship between the number of dresses owned by Sue and by Joan is shown in the diagram.



Step 2. Joan owns 28 dresses.

Joan—28

Step 3. Joan owns one more than three times as many dresses as Sue. So if one dress is taken from Sue, she will have exactly three times as many dresses as Joan.

$$\begin{aligned} \text{Joan's dresses} - 1 &= 3 \times \text{Sue's dresses} \\ 28 - 1 &= 3 \times \text{Sue's dresses} \\ 27 &= 3 \times \text{Sue's dresses} \end{aligned}$$

Step 4. Since 27 is three times as many dresses as Sue owns, the number of dresses owned by Sue can be obtained by dividing 27 by three.

$$\text{Sue's dresses} = \frac{27}{3} = 9$$

Step 5. Sue owns 9 dresses.

Problem 18

Jim's weekly income is $100.00 less than triple John's weekly income. Huey's weekly income is $20.00 more than double John's weekly income. Huey's income is $120.00. What is Jim's income?

Note: If you have difficulty you may find it helpful to follow these steps.

1. Diagram the relationship between John's and Huey's income.
2. Determine John's income.
3. Diagram the relationship between John's and Jim's income.
4. Determine Jim's income.

Original Problem

Jim's weekly income is \$100.00 less than triple John's weekly income. Huey's weekly income is \$20.00 more than double John's weekly income. Huey's income is \$120.00. What is Jim's income?

Problem Solution *(Diagrams are shown below)*

Step 1. The problem asks for Jim's income. It gives Huey's income as \$120.00 and it tells how to get John's income from Huey's income and then Jim's income from John's income.

Huey's income ⟶ John's income ⟶ Jim's income

Step 2. Huey's income is \$120.00. This is \$20.00 more than double John's income. So, if \$20.00 is subtracted, it will be double John's income. (See diagram)

$$\text{Huey's income} - \$20 = 2 \times \text{John's income}$$
$$\$120 - \$20 = \$100 = 2 \times \text{John's income}$$

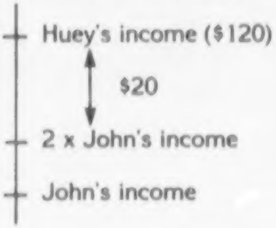
Step 3. Double John's income is \$100.00, so John's income is one-half of \$100.00.

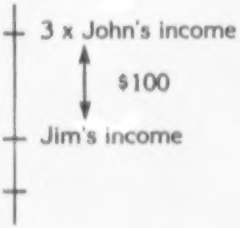
$$\text{John's income} = \frac{\$100}{2} = \$50$$

Step 4. Jim's income is \$100.00 less than triple John's weekly income. So triple John's income minus \$100.00 is Jim's income. (See diagram)

$$3 \times \text{John's income} - \$100 = \text{Jim's income}$$
$$3 \times \$50 - \$100 = \text{Jim's income}$$
$$\$150 - \$100 = \text{Jim's income}$$

Step 5. Jim's income is \$50.00.





Problem 19

The sum of the weekly income of Bill and his wife is $130.00 less than triple Judy's income. Bill makes $40.00 less than double what his wife makes. Bill's wife makes $80.00. How much does Judy make?

Original Problem

The sum of the weekly income of Bill and his wife is $130.00 less than triple Judy's income. Bill makes $40.00 less than double what his wife makes. Bill's wife makes $80.00. How much does Judy make?

Problem Solution

Step 1. Bill's wife makes $80.00.

Wife—$80

Step 2. Bill makes $40.00 less than double what his wife makes. So Bill's income can be obtained by doubling his wife's income and subtracting $40.00.

$$\begin{aligned}\text{Bill's income} &= 2 \times \text{wife's} - \$40\\ \text{Bill's income} &= 2 \times \$80 - \$40\\ \text{Bill's income} &= \$160 - \$40 = \$120\end{aligned}$$

Bill—$120

Step 3. The sum of Bill's and his wife's income is $130.00 less than triple Judy's income.

Step 4. The sum of Bill's and his wife's income is: $120 + $80 = $200.

Step 5. So $200.00 is $130.00 less than triple Judy's income. This means that if $130.00 is added to $200.00 we will have triple Judy's income.

$$\begin{aligned}\$130 + \$200 &= 3 \times \text{Judy's income}\\ \$330 &= 3 \times \text{Judy's income}\end{aligned}$$

Step 6. $330.00 is triple Judy's income. So Judy's income can be obtained by dividing $330.00 by 3.

$$\text{Judy's income} = \frac{\$330}{3} = \$110$$

Step 7. Judy's income is $110.00.

Problem 20

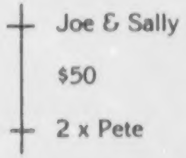
Double Pete's weekly income is $50.00 less than the combined incomes of Joe and Sally together. Patti, who makes $170.00 a week, makes $20.00 a week less than Sally but $40.00 more than Pete. How much does Joe make?

Original Problem

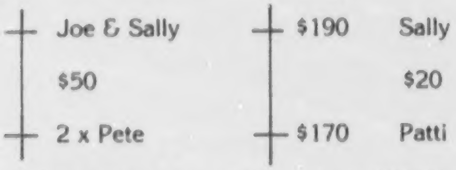
Double Pete's weekly income is $50.00 less than the combined incomes of Joe and Sally together. Patti, who makes $170.00 a week, makes $20.00 a week less than Sally but $40.00 more than Pete. How much does Joe make?

Problem Solution

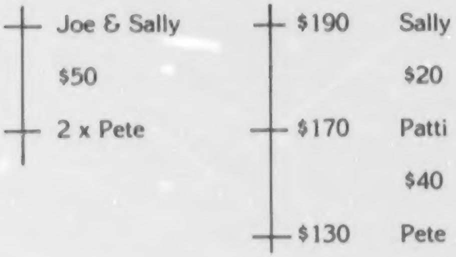
Step 1. The first sentence says that double Pete's income is $50.00 less than the combined incomes of Joe and Sally. This is shown in the following diagram.


Joe & Sally
$50
2 x Pete

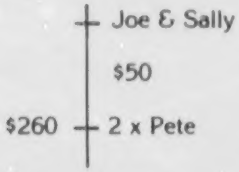
Step 2. The second sentence says Patti, who makes $170.00 a week, makes $20.00 a week less than Sally.

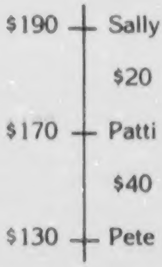

Joe & Sally
$50
2 x Pete
$190
Sally
$20
$170
Patti

Step 3. The sentence also says Patti makes $40.00 more than Pete.

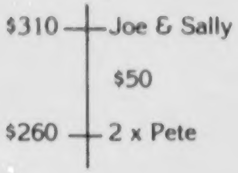

Joe & Sally
$50
2 x Pete
$190
Sally
$20
$170
Patti
$40
$130
Pete

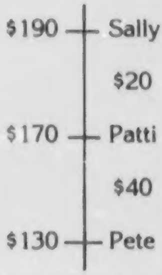
Step 4. If Pete's income is $130.00 a week, then double his income must be $260.00. This fact is placed on the left diagram.


Joe & Sally
$50
$260
2 x Pete


$190
Sally
$20
$170
Patti
$40
$130
Pete

Step 5. The diagram on the left now shows us that the combined incomes of Joe and Sally is $310.00.


$310
Joe & Sally
$50
$260
2 x Pete


$190
Sally
$20
$170
Patti
$40
$130
Pete

Step 6. We know that Sally makes $190.00 a week. Therefore Joe must make: $310 − $190 = $120 a week.

Problem 21

A stone statue was divided into 5 parts and packed in crates for shipping. The 5 full crates together weighed a total of 520 lbs, whereas each crate weighed 20 lbs empty. How much did the statue itself weigh?

Original Problem

A stone statue was divided into 5 parts and packed in crates for shipping. The 5 full crates together weighed a total of 520 lbs, whereas each crate weighed 20 lbs empty. How much did the statue itself weigh?

Problem Solution

Step 1. Each empty crate weighs 20 pounds. Therefore the weight of 5 crates is:

$$\begin{array}{r} 20 \\ \times\ 5 \\ \hline 100 \end{array}$$

Step 2. The weight of the statue is the total weight minus the weight of the 5 crates.

$$\begin{array}{r} 520 \\ -\ 100 \\ \hline 420 \end{array}$$

The statue weighs 420 pounds.

Problem 22

A metal statue was divided into 5 parts and packed in crates for shipping. Each full crate weighed 520 lbs, whereas it weighed 20 lbs empty. How much did the statue itself weigh?

Original Problem

A metal statue was divided into 5 parts and packed in crates for shipping. Each full crate weighed 520 lbs, whereas it weighed 20 lbs empty. How much did the statue itself weigh?

Problem Solution

Step 1. Each crate weighed 520 pounds full and 20 pounds empty, so the weight of the piece of statue in each crate was:

$$\begin{array}{r} 520 \\ -\quad 20 \\ \hline 500 \end{array}$$

Step 2. There were 5 pieces of statue each weighing 500 pounds, so the total weight of the statue was:

$$\begin{array}{r} 500 \\ \times\quad 5 \\ \hline 2500 \end{array}$$

Problem 23

Paul sold 160 sandwiches for $2.00 each. Each sandwich consisted of 4 oz of ham, 2 slices of bread, and mustard. Paul paid $3.00 a pound for the ham, 60¢ a loaf for the bread (20 slices per loaf) and used 8 jars of mustard at 50¢ each. How much profit did he make?

Original Problem

Paul sold 160 sandwiches for \$2.00 each. Each sandwich consisted of 4 oz of ham, 2 slices of bread, and mustard. Paul paid \$3.00 a pound for the ham, 60¢ a loaf for the bread (20 slices per loaf) and used 8 jars of mustard at 50¢ each. How much profit did he make?

Problem Solution

Step 1. 160 sandwiches require 320 slices of bread.

Step 2. There are 20 slices of bread in a loaf, so the number of loaves is:

$$\begin{array}{r} 16 \\ 20\,\overline{)320} \end{array}$$

Step 3. The cost of 16 loaves of bread is:

$$\begin{array}{r} 16 \\ \times\quad .60 \\ \hline 9.60 \end{array} \quad \text{or } \$9.60$$

Step 4. 160 sandwiches require 640 ounces of ham.

Step 5. There are 16 ounces in a pound, so the number of pounds of ham is:

$$\begin{array}{r} 40 \\ 16\,\overline{)640} \end{array}$$

Step 6. The cost of 40 pounds of ham is:

$$\begin{array}{r} 3.00 \\ \times\quad 40 \\ \hline 120.00 \end{array}$$

Step 7. The cost of 8 jars of mustard at 50¢ each is:

$$\begin{array}{r} .50 \\ \times\quad 8 \\ \hline 4.00 \end{array}$$

Step 8. The total cost for bread, ham and mustard is:

$$\begin{array}{rr} & 9.60 \\ & 120.00 \\ + & 4.00 \\ \hline & 133.60 \end{array}$$

Step 9. Paul sold 160 sandwiches for $2.00 each. So he collected:

$$\begin{array}{rr} & 160 \\ \times & 2.00 \\ \hline & 320.00 \end{array}$$

Step 10. Paul's profit is the amount of money he collected minus the total cost:

$$\begin{array}{rr} & 320.00 \\ - & 133.60 \\ \hline & 186.40 \end{array}$$

Paul's profit is $186.40.

Problem 24

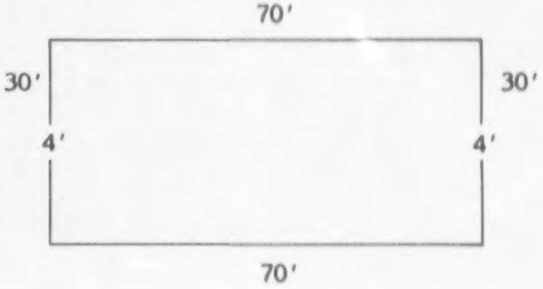
Ornamental chain sells for $1.23 a foot. How much will farmer Jones have to spend for chain in order to enclose a 70′ x 30′ patch of ground, leaving a 4′ entrance in the middle of each of the 30′ sides?

Original Problem

Ornamental chain sells for \$1.23 a foot. How much will farmer Jones have to spend for chain in order to enclose a 70′ x 30′ patch of ground, leaving a 4′ entrance in the middle of each of the 30′ sides?

Problem Solution

Step 1. Farmer Jones' garden is shown below with the 4′ entrances.



$$\begin{array}{r}70\\70\\30\\30\\-4\\-4\\\hline 192'\end{array}$$

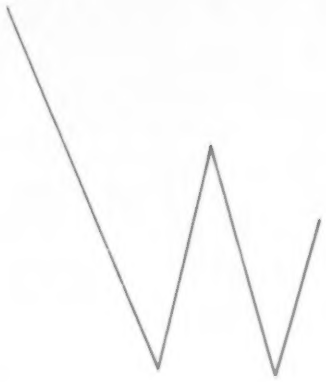
Step 2. The length of chain needed is 192 ft.
Therefore the cost is \$1.23 x 192 = \$236.16.

Note: The remaining problems require familiarity with some terms in mathematics and operations with fractions and powers. If you are unfamiliar with these topics but have developed the skill of careful, analytical thinking, you can learn them readily by taking a course in basic math in high school, college, or an adult education program. A good textbook in this area is *Developing Mathematical Skills: Computation, Problem Solving, and Basics for Algebra*, by Whimbey and Lochhead, McGraw Hill, Inc., 1981.

Problem 25

A certain ball, when dropped from any height, bounces one-third of the original height. If the ball is dropped from 54 ft, bounces back up, and continues to bounce up and down, what is the total distance that the ball has traveled when it hits the ground for the fourth time? Remember to count both the ascending and descending portions of the ball's path in computing the total distance.

Hint: make a diagram to represent the total path of the ball. You may want to show the ball bouncing at an angle, instead of straight up and down, in order to be able to see the ball's entire path. The beginning of such a diagram is shown below.



Original Problem

A certain ball, when dropped from any height, bounces one-third of the original height. If the ball is dropped from 54 ft, bounces back up, and continues to bounce up and down, what is the total distance that the ball has traveled when it hits the ground for the fourth time?

Problem Solution

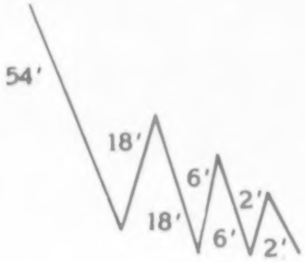
Step 1 Ball drops from a height of 54 ft. So when it hits the ground it will have traveled 54 ft.

Step 2. The ball bounces back up one-third of 54 ft: 54/3 = 18.

So, at that point it has travelled a total of 54 + 18 = 72 ft.

Step 3. The ball continues to fall and bounces up one-third as high.

The first 4 bounces are shown in the following diagram. (Although the bounces may have been straight up, they are drawn slanted to the left so that they can all be seen.)



Step 4. How far will it have traveled when it hits the ground for the fourth time?

$$\begin{array}{r} 54 \\ 18 \\ 18 \\ 6 \\ 6 \\ 2 \\ 2 \\ \hline 106 \end{array}$$

The sum is 106 ft.

Problem 26

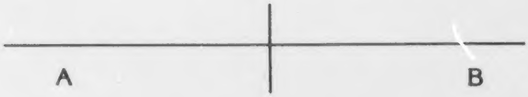
A car starts on a trip from city *A* to city *B* which is 60 miles away. It runs out of gas after it has gone one-third of the second half of the trip. How many miles still remain of the trip to city *B*?

Original Problem

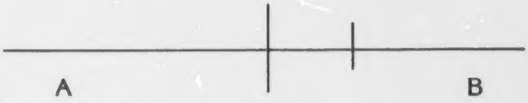
A car starts on a trip from city *A* to city *B* which is 60 miles away. It runs out of gas after it has gone one-third of the second half of the trip. How many miles still remain of the trip to city *B*?

Problem Solution

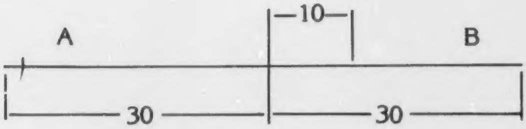
Step 1. The car ran out of gas after it went one-third of the second half of the trip. First the car went one-half of the trip. This is shown below.



Step 2. Then the car went one-third of the second half.



Step 3. The entire trip is 60 miles, so half the trip is 30 miles, and one-third of 30 miles is 10 miles.



Step 4. How many miles still remain of the trip to city B? There are 20 miles remaining to city B.

Problem 27

If the sum of two positive whole numbers is 10, what is their greatest possible product?

Note: When two numbers are multiplied together, the answer is called the "product".

Original Problem

If the sum of two positive whole numbers is 10, what is their greatest possible product?

Problem Solution

Step 1. The column labeled SUM shows all combinations of positive whole numbers whose sum is 10.

The column labeled PRODUCT shows the product for each combination.

SUM	PRODUCT
1 + 9 = 10	1 x 9 = 9
2 + 8 = 10	2 x 8 = 16
3 + 7 = 10	3 x 7 = 21
4 + 6 = 10	4 x 6 = 24
5 + 5 = 10	5 x 5 = 25

Step 2. The product is largest when the two numbers are equal:

5 x 5 = 25.

Problem 28

Is the sum of any two even positive integers—each of which is less than 10—odd or even?

Original Problem

Is the sum of any two even positive integers—each of which is less than 10—odd or even?

Problem Solution

Step 1. Here are the sums of all pairs of even positive integers, where each is less than 10:

$2 + 4 = 6$
$2 + 6 = 8$
$2 + 8 = 10$
$4 + 6 = 10$
$4 + 8 = 12$
$6 + 8 = 14$

Step 2. All these sums are even.

See appendix I for a more detailed discussion of this problem.

Problem 29

Which is bigger, *a* or *b*?

$a = 4^2 5^3$

$b = 4^3 5^2$

Note: 4^3 means 4 x 4 x 4

$4^4 5^2$ means 4 x 4 x 4 x 5 x 5

Original Problem

Which is bigger, *a* or *b*?

$a = 4^2 5^3$

$b = 4^3 5^2$

Problem Solution

Step 1. $a = 4 \times 4 \times 5 \times 5 \times 5$

$b = 4 \times 4 \times 4 \times 5 \times 5$

Step 2. These products can be rewritten like this:

$a = (4 \times 4 \times 5 \times 5) \times 5$

$b = (4 \times 4 \times 5 \times 5) \times 4$

Step 3. This shows that *a* is bigger.

Problem 30

Which is bigger, *a* or *b*?

$a = 2^5 4^3 7^6$

$b = 2^6 4^4 7^5$

Original Problem

Which is bigger, *a* or *b*?

$a = 2^5 4^3 7^6$

$b = 2^6 4^4 7^5$

Problem Solution

Step 1. Note that $7^6 = 7^5 \times 7$.

Therefore: $a = 2^5 4^3 7^6 = 2^5 4^3 7^5 \times 7$

Step 2. Note that $2^6 = 2 \times 2^5$ and $4^4 = 4 \times 4^3$.

Therefore: $b = 2^6 4^4 7^5 = 2 \times 2^5 \times 4 \times 4^3 \times 7^5 = 2^5 4^3 7^5 \times 2 \times 4$

Step 3. We now have: $a = 2^5 4^3 7^5 \times 7$

$b = 2^5 4^3 7^5 \times 2 \times 4$

or

$b = 2^5 4^3 7^5 \times 8$

Step 4. This shows that *b* is larger.

Problem 31

Here is how a number is multiplied by a fraction:

$$3 \times \frac{1}{35} = \frac{3 \times 1}{35} = \frac{3}{35}$$

$$3 \times \frac{2}{35} = \frac{3 \times 2}{35} = \frac{6}{35}$$

$$5 \times \frac{2}{35} = \frac{5 \times 2}{35} = \frac{\overset{1}{\cancel{5}} \times 2}{\underset{7}{\cancel{35}}} = \frac{2}{7}$$

Using this procedure, multiply 5^2 times $\frac{1}{4^7 5^3}$

Original Problem

Multiply 5^2 times $\frac{1}{4^7 5^3}$

Problem Solution

$$5^2 \times \frac{1}{4^7 5^3} = \frac{5^2}{4^7 5^3} = \frac{5 \times 5}{4^7 \times 5 \times 5 \times 5} = \frac{\overset{1}{\cancel{5}} \times \overset{1}{\cancel{5}}}{4^7 \times \underset{1}{\cancel{5}} \times \underset{1}{\cancel{5}} \times 5} = \frac{1}{4^7 5}$$

ADDITIONAL PROBLEMS

1. In the time a car travels 4 mi, a plane travels 30 mi. When the car travels 60 mi, how far does the plane travel?

2. Tarzan can swim 8 ft in the time a crocodile swims 10 ft. When Tarzan swims 20 ft, how far does the crocodile swim?

3. Paul can run 12 ft in the time Bob runs 10 ft.
   a. When Paul runs 6 ft, how far does Bob run? ________ ft
   b. When Paul runs 3 ft, how far does Bob run? ________ ft
   c. When Paul runs 30 ft, how far does Bob run? ________ ft
   d. When Paul runs 246 ft, how far does Bob run? ________ ft

4. A restaurant uses 2 pt of milk with 3 pt of cream to make coffee creamer. To make a larger quantity of the mixture, how many pints of cream should they use with 18 pt of milk?

5. A warped 12-in ruler is only 11 in long. Unaware of this, Judy used the ruler to measure off 48 in of rope. What was the true length of the rope?

6. A 5-in piece of cloth shrank to 4 in when it was washed. At that rate, how long would a 75-in piece of cloth measure after washing?

7. A clock runs fast. It says 8 min have passed when only 5 min have passed. How much time has really passed when the clock says 20 min have passed?

8. Yvette makes $20.00 a week less than double Celeste. Yvette makes $320.00 a week. How much does Celeste make?

9. Bob and Fred together make $20.00 a week less than double John. John makes $110.00 a week and Bob makes $140.00 a week. How much does Fred make?

10. Dero makes $70.00 a week more than double Willie. Also, Dero makes $15.00 a week less than Harry. Harry makes $185.00. How much does Willie make?

11. Beverly makes $25.00 less than Jody and Hal combined. Jody makes $30.00 less than Hal. Hal makes $140.00. How much does Beverly make?

12. If Joe's weekly income doubled he would be making $70.00 a week more than Joan. Joe's weekly income is $20.00 more than one-half of Bill's. Bill makes $200.00 a week. How much does Joan make?

13. If Tommy had three more books he would have twice as many as Helen. Tommy has 21 books. How many does Helen have?

14. The number of dresses owned by Beth is the number owned by Julie divided by the number owned by June. Julie, who owns 28 dresses, would own five times as many dresses as June if she had 7 more dresses. How many dresses does Beth have?

15. Jack weighs 25 lbs less than Phil. Bill weighs 40 lbs more than Jack. Frank weighs 70 lbs less than Bill. Al weighs 5 lbs less than Frank. Which man is heaviest? Which is lightest?

16. Starting at one end of my street, my house is the third house. Starting at the other end, my house is the seventh one. Not counting my house, how many houses are on my side of the street?

17. Starting at one end of a street, the police station is the fifth building. Starting at the other end, it is the ninth building. Including the police station, how many buildings are on that side of the street?

18. Fred and Tom attended the same school. Fred lives 7 mi away and Tom lives 9 mi away.

    a. What is the greatest possible distance between their homes?
    b. What is the shortest possible distance between their homes?

19. There are 15 books stacked in a single pile. How many books are between 2 other books?

Some of the following problems require familiarity with concepts and operations in mathematics such as fractions, decimals, percents, area, coordinates, negative numbers and powers. If you are unfamiliar with these topics but have developed the skill of careful, analytical thinking, you can learn them by taking a course in basic mathematics in high school, college or an adult education program. These problems are included here to show you the types of questions you may encounter on standardized academic aptitude tests.

20. A certain ball, when dropped from any height, bounces one-half the original height. If it was dropped from 80 ft and allowed to bounce freely, what was the total distance it traveled when it hit the ground for the third time?

21. A mouse made a dash from the cellar door to his hole in the living room wall which was 90 ft away. The cat saw him after he had gone one-third of the second half of the distance. How far did the mouse still have to run for safety? Make a diagram showing the distances.

22. A car started on a trip from city *X* to city *Y* which is 120 mi away. It ran out of gas one-quarter of the way through the last third of the trip. How many miles did it travel before running out of gas?

23. Is the sum of any two positive odd integers—each of which is less than 10—odd or even? (See appendix I for a detailed solution.)

24. Is the sum of any odd and even positive integers—each of which is less than 10—odd or even?

25. Is the sum of any 2 consecutive positive integers—each of which is less than 10—odd, even, or either?

26. Is the sum of any 3 consecutive positive integers—each of which is less than 10—odd, even, or either?

27. Is the sum of any 4 consecutive positive integers—each of which is less than 10—odd, even, or either?

28. Is the sum of any 5 consecutive positive integers—each of which is less than 10—odd, even, or either?

29. If the product of 2 positive equal numbers is 25, what is their sum?

30. A rectangular park is .6 mile long and .5 mile wide.

    a. What is the area of the park? (Area = Length x Width)
    b. What is the perimeter of the park? (The perimeter is the total distance around the park.)

31. A square park is 1.5 mi long on each side.

    a. What is the area of the park?
    b. What is the perimeter of the park?

32. If the area of a rectangular park is 30 square mi and the width of the park is .5 mi, what is the length?

33. If the perimeter of a square park is 1.2 square mi, what is the length of each side?

34. A square is 3 ft long on each side. A rectangle is 2 ft long and 7 ft wide. What is the ratio of the perimeter of the square to the perimeter of the rectangle?

35. One square is 6 in long on each side. Another square has sides which are only one-half as long. What is the ratio of the area of the small square to the large square?

36. Eighty-eight pounds of a medicine was divided into 1000 equal parts. How much did each part weigh?

37. Fred bought a car for $4000.00 and sold it 2 years later for 70% of what he paid.

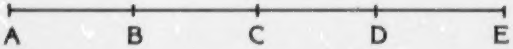
    a. How much did he sell it for?
    b. How much less did he sell it for than he paid for it?

38. A dealer bought a TV for \$300.00, then sold it for 40% more than that price.

   a. How much profit did he make?
   b. How much did he sell it for?

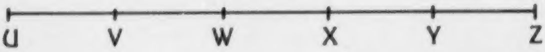
39. An art dealer bought a painting for \$500.00 and sold it 2 years later for 350% of the price he paid. How much did he sell it for?

40. The line below is marked off into 4 equal parts. If you start at point A and go 70% of the way to point E, between which 2 letters would you be? *Hint:* ¼ equals 25%.



41. a. The line below is marked off into 5 equal parts. If you start at point U and go 150% of the distance to point V, between which two letters would you be?

   b. If you start at point U and go 380% of the distance to point V, between which two letters would you be?



42. a. Which is bigger, *a* or *b*?

$a = 2^5 4^3 7^6$

$b = 2^6 4^4 7^5$

   b. Which is bigger, *a* or *b*?

$a = 2^7 3^8 7^9$

$b = 2^9 3^7 7^8$

43. Perform each multiplication and reduce the answer to simplest terms.

a. Multiply 20 times $\frac{1}{200}$

b. Multiply 30 times $\frac{1}{900}$

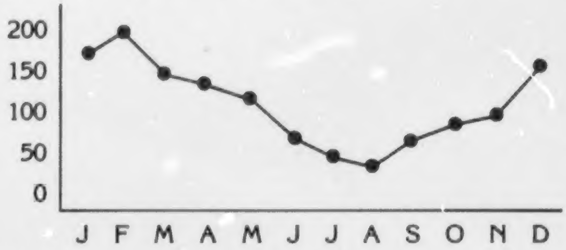
c. Multiply 5^3 times $\frac{1}{5^2}$

d. Multiply 5^2 times $\frac{1}{5^3}$

e. Multiply 5^3 times $\frac{1}{5^3}$

f. Multiply 5^3 times $\frac{1}{5^3 4^2}$

44. Monthly Utility Bills For The Jones In 1978

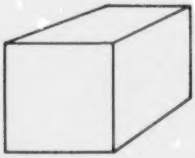


a. Is the average of the monthly utility bills more or less than \$150.00?

b. Is the average of the monthly utility bills for January and February more or less than the total of the bills for November and December?

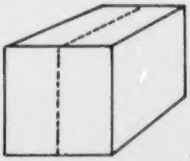
c. How much higher was the utility bill in January than in June?

45. a. How many faces does the cube shown below have?



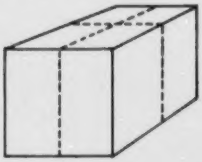
b. If each edge of the cube is 6 in long, how many square inches of paper are required to cover all faces of the cube?

46. a. If the cube below is cut in half along the dotted line, how many total faces do the two pieces have?



b. If each edge of the original cube was 6 in long, how many square inches of paper would be required to cover all the faces of the two pieces produced by cutting the cube?

47. a. If the cube below is cut into quarters along the dotted lines, how many total faces do the four pieces have?



b. If each edge of the original cube was 6 in long, how many square inches of paper would be required to cover all the faces of the four pieces produced by cutting the cube?

48. A company bought 60 gallons of soda for $30.00 and quart-size bottles for 5¢ each. How much total profit did they make if they sold all the soda for 50¢ a bottle? (One gallon equals 4 quarts.)

49. A garden is 50 ft long and 25 ft wide. How many total fence posts would be required to place a fence post at each corner and additional fence posts every 5 ft?

50. A field is 1500 ft long and 60 ft wide. How many total fence posts would be required to place a fence post at each corner and additional fence posts every 5 ft?

51. Mr. Smith's garden is 30 ft long and 21 ft wide and he would like to put a chain fence around it. The chain costs $5.00 a yard. He will support the chain by putting a metal post in each of the 4 corners and additional posts every 3 ft. The posts cost $2.00 each. What is the total cost of the chain and posts?

52. Starting at one corner, a park runs 30 mi east, 15 mi north, 10 mi east, 25 mi south, 40 mi west, then back to the first corner.

    a. Make a diagram of the park.

    b. Compute the distance around the park. (This distance is called the perimeter.)

53. Compute the area of the park described in problem 52 by dividing it into two rectangles and computing each part separately, then adding the two parts.

54. Starting at one corner, a park runs 10 mi west, 8 mi south, 20 mi west, 18 mi north, 30 mi east, then back to the first corner.

    a. Make a diagram of the park.

    b. Compute the distance around the park.

55. Compute the area of the park described in problem 54 by dividing it into two rectangles and computing each part separately, then adding the two parts.

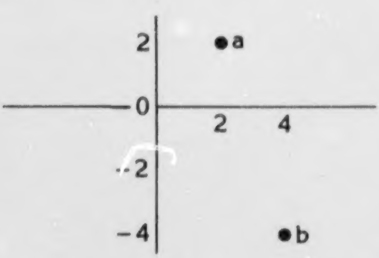
56. John had \$4000.00 in the bank on January 1, 1960. Each year for the next several years he spent 1/2 of the money he had in the bank at the beginning of the year. How much did he have in the bank on January 1, 1965?

57. Phil had \$540.00 in the bank on January 1, 1971. Each year for the next several years he spent 1/3 of the money he had in the bank at the beginning of the year. How much did he have in the bank on January 1, 1974?

58. Phil had \$540.00 in the bank on January 1, 1971. Each year for the next several years he spent 1/3 of the money he had in the bank at the beginning of the year. How much had he spent by January 1, 1974?

59. Jane deposited \$2000.00 in an account which paid 10% interest per year. At the end of the first year she left the original \$2000.00 and the interest in the account. How much was in the account at the end of the second year?

60.



a. What are the coordinates of the point *a*? What are the coordinates of the point *b*?

b. Compute the distance between points a and b using this formula:

$$\text{distance} = \sqrt{(x_b - x_a)^2 + (y_b - y_a)^2}$$

61. If x − 8 is a positive number and x is a whole number, what is the smallest possible value of x?

62. If x + 7 is a negative number and x is an integer, what is the largest possible value of x?

63. If $5(2 + 1) = 20 + x$, what is x?

64. If r is the radius of a circle and d is the diameter, which is larger:

$$r^2 \text{ or } \frac{d^2}{3}$$

65. If $x - 7 = y + 9$, which is larger, x or y?

XI. THE POST-WASI TEST

Introduction

If you have diligently worked through all the exercises in this book, your analytical reading and reasoning skills should be appreciably sharper. You have probably noticed this already while reading technical material or while solving problems. The test on the following pages is similar to the one you took at the beginning of the book. You will probably find, as do most people, that you can now deal with these questions more effectively and attain a higher score.

1. Which word is different from the other 3 words?

   a. water   *b.* ice   *c.* frozen   *d.* steam

2. Which letter is as far away from *P* in the alphabet as *M* is from *H*?

   a. J   *b. T*   *c. U*   *d. V*

3. If you are facing north and turn right, then make an about-face and turn right again, which direction is behind you?

4. Which pair of words fits best in the blanks?

   Bracelet is to wrist as ________ is to ________.

   a. neck : necklace
   b. painting : wall
   c. hair : ribbon
   d. jewelry : decoration

5. 3 is to 2 as 60 is to ________.

   a. 50   *b.* 20   *c.* 30   *d.* 40

6. Which set of letters is different from the other 3 sets?

   a. JKJI *b. GFGH* *c. POPQ* *d. NMNO*

7. In a different language *peq bo* means "green book," *sa bo mai* means "green old house," and *ja mai* means "old man." What is the word for house in this language?

8. Write the 2 letters which should appear next in the series.

   S T Q R O P M ___ ___

9. Marcia is taller than Phil but shorter than Jack. Regarding the occupations in which these people are engaged, the electrician is the shortest, the cashier is the tallest, and the accountant is intermediate. What is Marcia's occupation?

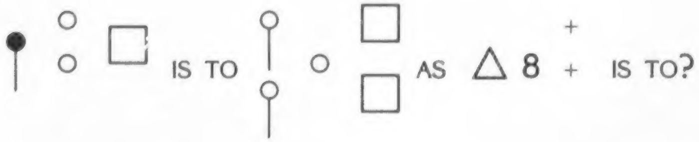
10. A car ran out of gas one third of the way through the second fifth of a 600-mile trip. How far had it traveled before running out of gas?

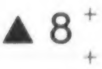
11. Which number in the following series is incorrect?

    12 8 15 11 17 14 21 17 24 20

    a. 11 *b.* 17 *c.* 18 *d.* 14

12. The first figure is related to the second figure in the same way that the third figure is related to one of the answer choices. Pick the answer.

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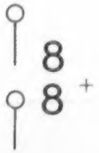
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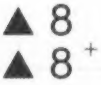
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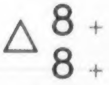
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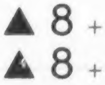
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13. Which pair of words best fits the meaning of the sentence?

The man was angry, ________ he was ________ with the child.

a. but—rough
b. but—gentle
c. so—gentle
d. nevertheless—rough

14. Write the 3 numbers which should appear next in the series.

284 280 140 136 68 64 32 28 14 ____ ____ ____

15. A cardiologist is a ________ specialist.

a. brain *b.* heart *c.* ear and throat *d.* lung *e.* bone

16. Calligraphy pertains to:

a. minerals *b.* penmanship
c. supernatural *d.* Egypt

17. Pat makes $30.00 less than Jim and Mike combined. Pat makes $500.00 and Mike makes $210.00. How much does Jim make?

18. Which letter is in the same position in the alphabet as the letter *s* is in *comprehension*?

a. c *b.* j *c.* a *d.* i *e.* none of the above

19. A tower always has __________.

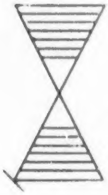
a. doors *b.* windows *c.* metal *d.* width

20. Circle the letter before the letter in the word diction which occupies the same position in the word as it does in the name of the ninth month.

21. Which pair of words is different from the other 3 pairs?

a. talk—fast *b.* smile—laugh
c. eat—heartily *d.* look—carefully

22. The top 4 figures form a series which changes in a systematic manner according to some rule. Try to discover the rule and choose from among the alternatives the figure which should occur next in the series.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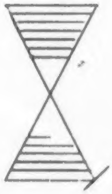
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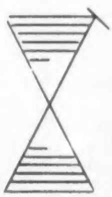
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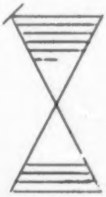
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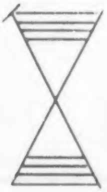
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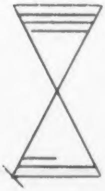
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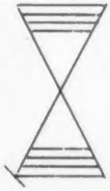
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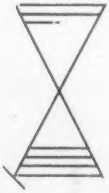
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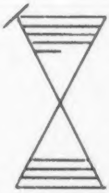
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23. Which number is repeated first in the following series?

6 3 8 1 2 9 7 4 5 8 5 1 3 7 9 4 2 4

24. Which pair of words fits best in the blanks?

Room is to door as _________ is to _________.

a. entrance : exit
b. stairs : building
c. bottle : cap
d. rug : floor

25. Write the 3 pairs of letters which should come next.

cY eV gW iT kU mR oS qP ____ ____ ____

26. One-half is to 8 as 5 is to _________.

a. 20 *b.* 10 c. 75 *d.* 80

27. Forest is to tree as _________ is to _________.

a. family : parents *b.* plant : roots
c. tree : leaves *d.* tribe : indian

28. Which word means the opposite of *ostracize*?

a. expel *b.* welcome c. point *d.* relevant

29. Which set of letters is different from the other 3 sets?

a. *AJKC* b. *LPQN* c. *GBCH* d. *SIJU*

30. Umbrella is to wet as _________ is to _________.

a. sunshine : rain *b.* coat : warm
c. napkin : dirty *d.* boots : feet

31. A car travels 30 miles in the time a boat travels 20 miles. How far does the car travel when the boat travels 90 miles?

a. 60 *b.* 100 c. 135 *d.* 125

32. Alloy is to iron as __________ is to __________.

a. metal : steel
b. chord : middle C
c. B flat : C sharp
d. copper : bronze

33. How many fifths are there in 45/3?

a. 3 *b.* 1 *c.* 60 *d.* 75

34. If 3 days before tomorrow is Friday, what is the day after tomorrow?

____

35. Which word is different from the other 3 words?

a. pernicious *b.* noxious *c.* palliative *d.* detrimental

36. 5 9 3 8 4 1 2 6. If the fourth number is larger than the seventh number, add the second number to the fifth number; otherwise add the first number of the eighth number. Either way, add the second number to your sum unless the seventh number is smaller than the third number. In that case add the fifth number. Write your answer here.

____

37. Select the answer which is most nearly equivalent in meaning to the following staternent.

Wise men learn more from fools than fools from the wise.

a. Knaves and fools divide the world.

b. It is easier to be wise for others than for ourselves.

c. He gains wisdom in a happy way, who gains it by another's experience.

d. Accuracy of statement is one of the elements of truth; inaccuracy is a near kin to falsehood.

38. Nostalgia is to anticipation as ________ is to ________.

a. present : past
b. future : past
c. past : future
d. future : present

After you have completed the test and checked your answers, you may want to go back and rework problems in the book which are similar to any that you missed.

APPENDIX I

New knowledge in mathematics often results from a mathematician seeing a consistency and then showing theoretically why the consistency occurs. The process is similar to the steps you might use in solving this problem.

Is the sum of any two even integers odd or even?

Solution

Step 1. We will try adding several pairs of even integers to see if the sums are odd or even. We attempt to avoid using only special cases for which a conclusion might be true even though it is not true for all cases.

Experimenting with a few concrete cases is a good way to start many math problems involving general formulas or statements.

$$2 + 2 = 4 \qquad 2 + 18 = 20 \qquad 64 + 100 = 164$$

$$-10 + 16 = 6 \qquad -18 + -50 = -68 \qquad 0 + 14 = 14$$

Step 2. All these sums are even numbers, so it appears that the sum of any two even integers is even.

Step 3. To be sure that the sum of any pair of even integers is even, we will begin with the definition of an even integer.

When an even integer is divided by 2 the answer is an integer, not a fraction or a mixed number.

This means that if A and B are any even integers, then:

$\frac{A}{2}$ is an integer $\qquad$ $\frac{B}{2}$ is an integer

Step 4. We want to show that if A and B are even integers, then their sum (A + B) is also an even integer.

This means we must show that $\frac{A + B}{2}$ is an integer, not a fraction or a mixed number.

Step 5. From the rule for adding fractions we know that:

$$\frac{A+B}{2} = \frac{A}{2} + \frac{B}{2}$$

Step 6. We know that $\frac{A}{2}$ and $\frac{B}{2}$ are both integers.

Also, the sum of two integers is itself an integer.

Therefore, $\frac{A+B}{2}$ is an integer.

Step 7. This shows that the sum of any two even integers is also an even integer.

Here are some other problems that could be approached in the same way. Just remember that when 1 is divided by 2 the answer is ½, when −1 is divided by 2 the answer is −½, and when other odd numbers are divided by 2 the answer is a mixed number with the fraction portion equal to ½. Furthermore, ½ + ½ = 1.

Is the sum of any two odd integers odd or even?

Is the sum of any odd and even integer odd or even?

Is the sum of any two consecutive integers odd or even?

Is the sum of any three consecutive integers odd, even, or either?

APPENDIX II. ANSWER KEY

Whimbey Analytical Skills Inventory (WASI)

1. c
2. c
3. a
4. c
5. d
6. c
7. a
8. O, N
9. d
10. c
11. d
12. d
13. c
14. 87, 83
15. e
16. e
17. d
18. pardøn
19. c
20. c
21. c
22. d
23. d
24. a
25. Q, Q, G
26. d
27. d
28. b
29. b
30. d
31. d
32. b
33. c
34. stop
35. c
36. b
37. c
38. b

Chapter IV. Additional Problems

1. Sally is taller.
2. John is fastest; Harvey is slowest.
3. Fred is tallest; Hal is second tallest.
4. Gladys is heaviest; Violet is lightest.
5. more hatred

   Frankenstein
   Mummy
   Wolfman
   Dracula

   less hatred

6. more

   Instruction
   Operation, Maintenance and Auxiliary Agencies
   Capital Outlay
   Interest
   General Control

   less

7. deeper

   Superior
   Michigan
   Ontario
   Huron
   Erie

   shallower

8. $350.00

9.

	Cotton	Wool	Synthetic	Total
Pat	3	4	3	10
Joan	8	3	1	12
Mary	4	3	1	8
Total	15	10	5	30

Joan has 12 dresses.

10. 4 handguns

11.

	First Quarter	Second Quarter	Third Quarter	Fourth Quarter	Total
Acme	23	7	4	23	57
B&B	0	50	0	25	75
Arco	0	0	75	0	75
Total	23	57	79	48	207

A total of 57 houses was sold in the second quarter.

12. Nancy

13. Judy—Housewife; Celia—Math Teacher; Betty—Truck Driver

14. Rose—Saleswoman; Hannah—Shipping Clerk; Geraldine—Cashier; Mary Jo—Stockgirl

15. West

16. Northwest

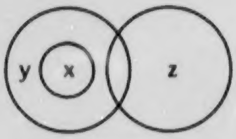
17. 5 hours and 45 minutes

18. East-West

19. Parallel

20. Perpendicular

21.



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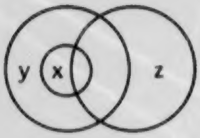

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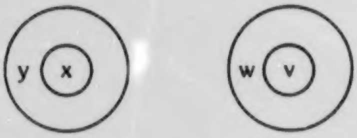

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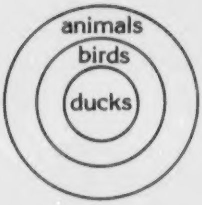
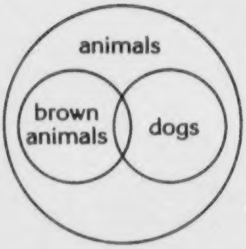
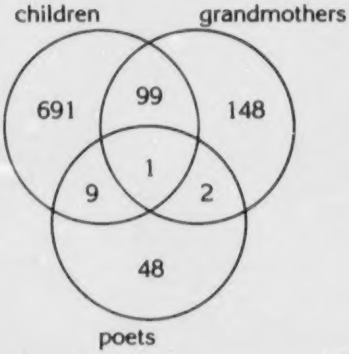

animals
birds
ducks

25.

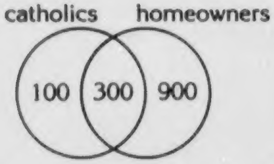

animals
brown animals
dogs

26.


children
grandmothers
691
99
148
1
9
2
48
poets

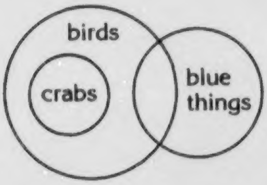
Total Homes = 998

27.

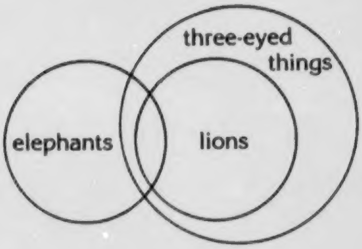

catholics
homeowners
100
300
900

Total votes = 1300

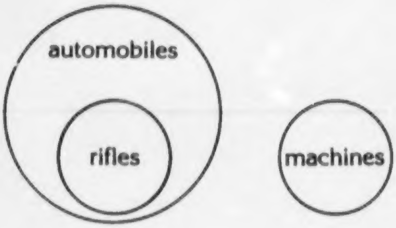
28. *a.* no
b. no


birds
crabs
blue things

29. *a.* no
b. yes

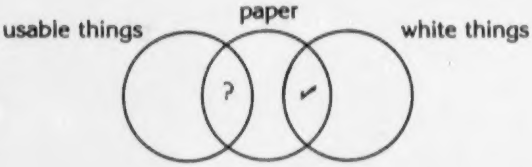

three-eyed things
elephants
lions

30. a. no
    b. yes



31. a. No. The question mark shows we don't know whether some usable things are paper.

    b. Yes. The ✓ shows that some paper is white and therefore not usable.



32. fi~~r~~eplace

33. Otherwise, circle the first word in this sentence.

34. If deleting the first, third, fifth and seventh letters . . .

35. Joanth~~a~~n

36. sanctuary

37. *8 9 7 5 ③ 0 9 2 4 6*

38. *9 8 7 6 5 4 3 2 1*. Take the difference between the first number and the sixth number. Write the difference here 5. Now take the difference between the fifth number and the seventh number. Write it here 2. Finally, take the difference between these two differences and write it here 3.

39. *8②7 5 6 4 5 3 4.*

40. c. 10

41. *gi ra*

42. c. *ba si*

Chapter VI. Analogy Problems

1. a
2. c
3. b
4. c
5. b
6. c
7. b
8. c
9. a
10. a
11. b
12. a
13. b
14. b
15. c
16. b
17. c
18. b
19. a
20. c
21. a
22. b
23. a
24. b

Chapter VII. Samples of Acceptable Relationship Sentences for Relationship Problems

1. ______ is the opposite of ______ .

2. ______ are kept in a ______ .

3. A ______ was used before a(n) ______, but for the same purpose.

4. A ______ controls the action of a(n) ______ .

5. A(n) ______ gives birth to a ______ .

6. ______ energizes a ______ .

7. An ______ needs ______ .

8. The ______ provide(s) nutrition for a(n) ______ .

9. The ______ is at the end of the limb which is attached to the body by the ______ .

10. A ______ is a type of ______ .
11. A ______ is given by a ______ .
12. A ______ hunts a ______ .
13. ______ is three times ______ .
14. ______ is 30 less than ______ .
15. ______ is ⅓ of ______ .
16. ______ is 30 more than ______ .
17. ______ is 2½ times ______ .
18. (A) ______ are at the base of a ______ .
19. The ______ revolves around the ______ .
20. ______ travel together in a ______ .
21. A ______ is used to produce ______ .
22. ______ is a mild form of ______ .
23. ______ is the extreme of ______ .
24. The purpose of a(n) ______ is to counter ______ .
25. A ______ is a three-dimensional ______ .
26. A ______ is a signal before a ______ .
27. A(n) ______ is the outline of a ______ .
28. ______ can be used to ______ .
29. A ______ extends into the ______ .
30. A(n) ______ is the origin of a(n) ______ .
31. A ______ may be the light source of a ______ .
32. ______ is an inactive state of a(n) ______ .
33. ______ means ______ .

Chapter VIII.

Section	Answer	Relationship Sentence
5	c	______ is ⅓ of ______ .
6	d	A ______ is run by a ______ .
7	c	A(n) ______ is a collection of ______ .
10	c	______ is in the form of a ______ .
13	d	______ is the natural covering of a ______ .
14	c	A ______ is part human and part ______ .

Chapter VIII. Analogy Problems

	Answer	Relationship Sentence
1.	b	A ______ stands on a ______ .
2.	c	A ______ is used for hitting in ______ .
3.	d	(A) ______ is produced at a ______ .
4.	c	A ______ is a unit of ______ .
5.	d	______ is three months before ______ .
6.	c	______ make up a(n) ______ .
7.	d	______ is destructive to ______ .
8.	d	______ is the meat obtained from ______ .
9.	d	A ______ extends from the ______ .
10.	b	A ______ is part of a(n) ______ .
11.	a	A ______ produces ______ .
12.	c	The product of a ______ is a ______ .
13.	c	A ______ is used to measure ______ .
14.	d	______ is the reward of ______ .
15.	b	A ______ sells high ______ food.
16.	c	A ______ fights ______ .
17.	c	______ and ______ sound the same.
18.	c	A ______ may be made of ______ .
19.	a	One part of a ______ is a ______ .
20.	b	______ stimulates the growth of ______ .
21.	a	A ______ is bigger than but similar to a ______ .
22.	a	A(n) ______ may give rise to a(n) ______ .
23.	b	A(n) ______ is (can be) surrounded by (the) ______ .
24.	b	______ is a ______ .

Chapter IX. Problems in Identifying Patterns

1. *42 47 50*

   Pattern description: Alternately add 5 and 3.

   or

   Each number is 8 more than two numbers before it.

2. *A B B*

   Pattern description: A single *A* alternates with alternately one or two *B*s.

3. *g*

   Pattern description: Numbers running backwards alternate with letters that go up the alphabet skipping a letter each time.

4. *17 20 19*

   Pattern description: Alternately add 3 and subtract 1.

   or

   Each number is 2 more than two numbers before it.

5. *J I L*

   Pattern description: Go down one letter, then up three letters, repeated.

   or

   Letters go up the alphabet in sets of two, with the two letters within each set in reverse order.

6. *Q L Q*

   Pattern description: *Q*s alternate with *L*s. With the *Q*s there are alternately 2 or 4. With the *L*s there are alternately 1 or 3.

7. *7 4 2*

   Pattern description: Alternately subtract 3 and subtract 2.

   or

   Each new number is 5 less than two numbers before it.

8. *W E V*

Pattern description: Letters going up the alphabet alternate with letters going down the alphabet.

9. *18 20 15*

Pattern description: Alternately subtract 5 and add 2.

or

Each new number is 3 less than two numbers before it.

10. *121212 1212121*

Pattern description: Each set becomes longer by alternately adding 1 or 2.

11. *43 52 62*

Pattern description: Each time add 1 more than was previously added.

12. *129 126 119*

Pattern description: Subtract 3, subtract 7, add 4, then repeat.

or

Each number is 6 less than three numbers before it.

13. *T I R*

Pattern description: There are two alternating series of letters. One series skips a letter going up the alphabet. The other series skips a letter coming down the alphabet.

14. *P 5 M*

Pattern description: The letters *J*, *P* and *M* alternate with the numbers 1, 3, 5 and 8.

15. *5 1 2 C P*

Pattern description: Each letter is followed by a number showing its position in the alphabet.

16. *320 640 1280*

Pattern description: Each new number is twice the previous number.

17. *38H 46F*

Pattern description: Numbers alternate with letters. Each new number is obtained by adding 1 more than was added in getting the previous number. The letters run backwards in the alphabet, each time skipping a letter.

18. *85 75 64*

Pattern description: Each new number is obtained by subtracting 1 more than was subtracted in getting the previous number.

19. *B 2 A B*

Pattern description: Groups of letters alternate with groups of numbers. The letters are *A B C B A*. The numbers are *1 2 3 2 1*.

20. *M M 13 B M*

Pattern description: Groups of letters alternate with groups of numbers. The letters are *B M N M B*. The numbers are *2 13 14 13 2*.

21. *486 1458*

Pattern description: Each new number is three times the previous number.

22. *3F F4 3E G2*

Pattern description: Each entry consists of a letter and a number and begins alternately with one or the other. The numbers are *2 3 4 3 2 3 4 3 2 3* etc. The letters start with *A* and go up one letter, repeat that letter, then go down one letter. Next jump up two letters, go up one letter, repeat that letter, go down one letter. Next jump up two letters, etc.

23. *K P P*

Pattern description: Each letter is repeated once; then the next letter is obtained by going up the alphabet one more letter than the last time.

24. *I K I*

Pattern description: Up 2 letters, up 2 letters, down 2 letters, repeated.

25. *2 1 ½ ¼*

Pattern description: Each new number is one-half the previous number.

26. *11 K 11 1 A A*

Pattern description: Sets of numbers alternate with sets of letters. The numbers are *1 11 20 11 1.* The letters are *A K T K A.*

27. *ACEG GACE*

Pattern description: Starting with the letters *ACEG*, each successive entry has the letter from the back of the previous entry moved to the front.

28. *000X0X0 0000X0X*

Pattern description: In each entry the *X*s are one position to the right compared to the previous entry.

29. *34 33 66*

Pattern description: Alternately multiply by 2 and subtract 1.

30. *28 21 13*

Pattern description: Starting with the number 49, first 1 is subtracted, then 2 is subtracted, then 3 is subtracted, and in each case 1 more is subtracted than was subtracted last.

31. *4 13 3*

Pattern description: Starting with 8, first 1 is added, then 2 is subtracted, then 3 is added, and this pattern of alternately adding and subtracting 1 more number is continued.

32. *38 76 78*

Pattern description: Alternately add 2 and multiply by 2.

33. *24FP 26GV*

Pattern description: Each entry has a number and two letters. The numbers are obtained by alternately adding 2 and 5. The first letter in each entry is one letter higher in the alphabet than the first letter in the previous entry. The second letter in each entry is obtained by skipping one more letter than was skipped in getting the second letter of the previous entry.

34. *T W V*

Pattern description: Up 3 letters, down 1 letter, up 2 letters, repeated.

35. *720 40,320*

Pattern description: Each entry is multiplied by 1 more than the previous entry was multiplied by.

36. *17 21*

Pattern description: The third number in each set is the sum of the first two numbers.

37. *9 10 21*

Pattern description: The third number in each set is the sum of the first two numbers.

38. *34 55 89*

Pattern description: Each number is the sum of the two previous numbers.

39. *68 125 230*

Pattern description: Each number is the sum of the three previous numbers.

40. *8700 8600 9600*

Pattern description: Alternately add 1000 and subtract 100.

41. *31 36 38*

Pattern description: Add 5, add 2, subtract 1, then repeat.

42. *70 80 87*

Pattern description: Add 10, add 7, add 5, then repeat.

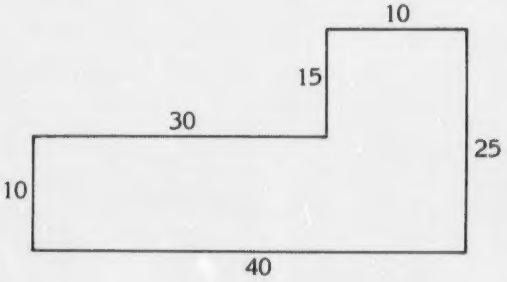
43. *18 19 38*

    Pattern description: Subtract 4, add 1, multiply by 2, then repeat.

Chapter X. Additional Problems

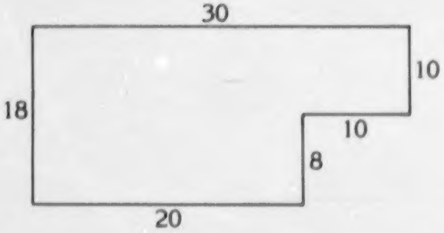
1. 450 mi
2. 25 ft
3. *a.* 5
   b. 2½
   c. 25
   d. 205
4. 27
5. 44 in
6. 60 in
7. 12½ min
8. $170.00
9. $60.00
10. $50.00
11. $225.00
12. $170.00
13. 12
14. 4
15. Heaviest—Bill
    Lightest—Al
16. 8
17. 13
18. *a.* 16
    b. 2
19. 13
20. 200 ft
21. 30 ft
22. 90 mi
23. even
24. odd
25. odd
26. either
27. even
28. either
29. 10
30. *a.* .3 sq mi
    b. 2.2 mi
31. *a.* 2.25 sq mi
    b. 6 mi
32. 60 mi
33. .3 mi
34. 12/18 or 2/3

35. 9/36 or 1/4
36. .088
37. *a.* $2800.00
    b. $1200.00
38. *a.* $120.00
    b. $420.00
39. $1750.00
40. C and D
41. *a.* V and W
    b. X and Y
42. *a.* b
    b. b
43. *a.* 1/10
    b. 1/30
    c. 5
    d. 1/5
    e. 1
    f. $1/4^2$
44. *a.* less
    b. less
    c. $100.00
45. *a.* 6
    b. 216 sq in
46. *a.* 12
    b. 288 sq in
47. *a.* 24
    b. 360
48. $78.00
49. 30
50. 208
51. $238
52. *a.*



b. 130 mi

53. 550 sq mi
54. *a.*



b. 96 mi

55. 460 sq mi
56. $125.00
57. $160.00
58. $380.00
59. $2420.00
60. (2, 2) (4, −4)
distance = $\sqrt{40} = 6.32$
61. 9
62. −8
63. −5
64. $\frac{d^2}{3}$
65. x

Chapter XI. The Post-WASI Test

1. C
2. C
3. south
4. B
5. D
6. A
7. sa
8. N K
9. accountant
10. 160
11. B
12. C
13. B
14. 10, 5, 1
15. B
16. B

17 $320

18. B
19. D
20. DIⒸTION
21. B
22. C
23. 8
24. C
25. sQ, uN, wO
26. D
27. D
28. B
29. C
30. C
31. C
32. B
33. D
34. Tuesday
35. C
36. 17
37. C
38. C

APPENDIX III

Compute Your Own IQ

You can use the following table to approximate your IQ from your WASI score. Your IQ can vary, depending upon your health and psychological condition. So don't regard your computed IQ as a fixed quantity. If you scored 35 or more on the WASI you should have no trouble with advanced high school and college work, assuming you are willing to devote the necessary time and effort to it. But if you scored below 35, the exercises in this book will be extremely useful in improving your academic aptitude and computed IQ.

Students below age 17 are expected to score lower on the WASI. For students between 14—16, the IQ can be approximated by using the appropriate table.

The IQ of students below 14 cannot be accurately estimated from the WASI, but the exercises in the book can still be used to improve their analytical reasoning skills.

A full explanation of the history and limitations of IQ measurement can be found in *Intelligence Can Be Taught* by Arthur Whimbey.

Adult: 17 and Older		*WASI Scores for Younger Age Groups*		
WASI Score	*IQ Score*	*Age 16*	*Age 15*	*Age 14*
Over 35 →	Over 125 ←	Over 34	Over 33	Over 32
33—35	122—125	32—34	31—33	30—32
31—32	119—121	30—31	29—30	28—29
29—30	116—118	28—29	27—28	26—27
27—28	113—115	26—27	25—26	24—25
25—26	110—112	24—25	23—24	22—23
23—24	107—109	22—23	21—22	20—21
21—22	104—106	20—21	19—20	18—19
19—20	101—103	18—19	17—18	16—17
17—18	98—100	16—17	15—16	14—15
15—16	95—97	14—15	13—14	12—13
Below 15	Below 95	Below 14	Below 13	Below 12

PROBLEM SOLVING AND COMPREHENSION

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ABOUT THE AUTHORS

Arthur Whimbey is a cognitive psychologist and is the author of *Intelligence Can Be Taught* and (with Jack Lochhead) *Developing Mathematical Skills.* Jack Lochhead is Assistant Dean at the University of Massachusetts, Amherst, and Director of the Cognitive Development Project at the same university. In 1978 he organized a conference on Cognitive Process Instruction, resulting in the book of that name, for which he was editor. Both Drs. Whimbey and Lochhead give frequent workshops on problem solving.

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